PROCEEDINGS OF ANNUAL MEETING ON THEORETICAL ASPECTS OF CONTROLLED THERMONUCLEAR RESEARCH

April 7-9, 1976
University of Wisconsin—Madison
Madison, Wisconsin

Proceedings of the

Annual Meeting on Theoretical Aspects of

Controlled Thermonuclear Research

April 7-9, 1976

Sponsored by the

University of Wisconsin - Madison

Madison, Wisconsin 53706

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GENERAL INFORMATION

1976 ANNUAL MEETING ON THEORETICAL ASPECTS OF CTR

The morning sessions all begin at 9:00 AM, and are held in the Lakeshore Room. Coffee and various other refreshments will be available from 10:00 to 11:30 AM each morning and from 2:30 to 4:30 PM each afternoon. If necessary, closed circuit television coverage of the morning sessions will be available in room 138 of the Wisconsin Center.

Afternoon poster sessions will take place, beginning at 2:00 PM in both the Blue Lounge and the Alumni Lounge. In order to minimize the requirement of each author having to remain at his display throughout the session, the sessions will be divided into two groups. Session 1 will be in the Alumni Lounge, while Session 2 will be in the Blue Lounge. It would be appreciated if the authors of both sessions would arrange to have their display posted at the 2:00 PM starting time. However, session one will be officially open from 2-3:30 PM, and session two from 3:30-5:00 PM.

The traditional buffet will be held on Thursday evening, April 8th beginning at 6:30 PM, in the Alumni Lounge with a cocktail bar hosted by the University of Wisconsin held concurrently. Admission to the buffet will be by badge only.

Registration will be open in the lobby of the Wisconsin Center from 6-8:30 PM on Tuesday, April 6th, and beginning at 8:15 AM on the morning of April 7th. During the conference, in addition to the Lakeshore Room and the Alumni Lounge, the Blue Lounge, and room 138 will be available for conversations and study.

You will also note that the conference proceedings book is organized by session, rather than alphabetically as previously.

For those wishing to visit some of the plasma research facilities on the campus, octupole, stellarator, etc., an open house is arranged for Wednesday evening. Details will be on the bulletin board at the registration desk.

SCHEDULE

1976 ANNUAL MEETING ON THEORETICAL ASPECTS OF CTR

WEDNESDAY, 7 April

9:00 AM	Morning Session, Lakeshore room, H. L. Berk
10:00-11:30	Coffee
12:30	Lunch
2:00 PM	Poster Session 1A, Alumni Lounge
2:30 - 4:30	Coffee
3:30	Poster Session 2A, Blue Lounge
5:00	Afternoon Session Adjurns

THURSDAY, 8 April

9:00 AM	Morning Session, Lakeshore room, A. Bers
10:00-11:30	Coffee
12:30	Lunch
2:00 PM	Poster Session 1B, Alumni Lounge
2:30 - 4:30	Coffee
3:30	Poster Session 2B, Blue Lounge
5:00	Afternoon Session Adjurns
6:30	Buffet Dinner and Cocktails, Alumni Lounge

FRIDAY, 9 April

9:00 AM	Morning Session, Lakeshore room, J. P. Freidberg
10:00-11:30	Coffee
11:00	Lakeshore room, C. L. Hedrick
12:30	Lunch
2:00 PM	Poster Session 1C, Alumni Lounge
2:30 - 4:30	Coffee
3:30	Poster Session 2C, Blue Lounge
5:00	Afternoon Session Adjurns

PROGRAM

1976 ANNUAL MEETING ON THEORETICAL ASPECTS OF CTR

WEDNESDAY, 7 April

9:00 AM WELCOME ADDRESS - Robert M. Bock, Dean
University of Wisconsin-Madison
Graduate School

PLENARY SESSION - Chairman, J. L. Shohet

P1 9:05 AM H.L. Berk
Quasilinear Calculations of Plasmas in Mirror Machines

P2 9:45 AM J.A. Byers
Simulation of High β Plasmas in Mirror Machines

P3 10:10 AM R.N Sudan and M.N. Rosenbluth
Low-Frequency Stability of Field Reversed Ion Rings

P4 D.C. Stevens and E. Turkel
Diffusive Transition from Belt Pinch to Doublet

P5 M.S. Chu, R.L. Miller and T.H. Jensen
Axisymmetric Stability in Tokamaks

P6 R.B. White, D.A. Monticello, M.N. Rosenbluth and B.V. Waddell
Saturation of the Tearing Mode

P7 H.R. Strauss
Tokamak Sawtooth Oscillations Due to Tearing Modes

P8 O. Betancourt and P. Garabedian
Equilibrium and Stability Code for a Diffuse Plasma in Three Dimensions

POSTER SESSION 1A - MHD STABILITY THEORY

1A-1 2:00 PM B. Basu and B. Coppi
"Kinetic" Theory of Internal Kink Modes

1A-2 L. Chen, J.M. Finn, P.K. Kaw and P.H. Rutherford
On Drift-Tearing and Driven Tearing Modes

1A-3 J.Y. Choe
Instabilities of Straight Cylindrical Anisotropic Guiding Center Plasma

1A-4 B. Coppi, R. Galvao, M. Rosenbluth and P. Rutherford
Resistive Internal Kink Modes

1A-5 D.R. Dobrott
The Local Stability of Doublet

MHD STABILITY THEORY (Continued)

1A-6 R. Gruber, D. Berger and F. Troyon
Finite Hybrid Elements to Compute the Entire Ideal 2D MHD Spectrum

1A-7 E. Hameiri
Spectrum and Local Stability of MHD Equilibria with Flow

1A-8 H.R. Hicks, J.W. Wooten and G. Bateman
Nonlinear Ideal MHD Evolution: Verification

1A-9 S.C. Jardin, J.L. Johnson, J.M. Greene, M.D. Rosen and R.C. Grimm
Two-Dimensional Time-Dependent Calculations of Axisymmetric Equilibrium and Stability of Takamaks

1A-10 W. Kerner
Numerical Study of the MHD Spectrum in Tokamaks With A Noncircular Cross Section

1A-11 G. Laval, M.N. Rosenbluth and B.V. Waddell
Influence of Finite Gyration Radius on the m=1 Internal Mode in Tokamaks

1A-12 J. Manickam, M.S. Chance, R.C. Grimm, J.M. Greene and J.L. Johnson
The Princeton Equilibrium and Stability Package

1A-13 D.A. Monticello, R.B. White, B.V. Waddell and M.N. Rosenbluth
Nonlinear Growth of the Tearing Mode During the Skin Current Phase of the Tokamak Discharge

1A-14 D.B. Nelson
Effect of Hot Electrons on Localized MHD Instabilities

1A-15 A. Salat and E. Rebhan
Axisymmetric MHD Instabilities of Doublet-type Plasma Configurations

1A-16 G.O. Spies and D.B. Nelson
Stability of the Elmo Bumpy Torus

1A-17 L. Turner
The Sharp-Boundary Screw Pinch: Kinetic Corrections To Magnetohydrodynamics According to Vlasov-Fluid Theory

1A-18 B.V. Waddell, M.N. Rosenbluth, D.A. Monticello and R.B. White
Nonlinear Growth of the m=1 Tearing Mode

1A-19 C.E. Wagner, N.A. Krall and R.C. Davidson
Simulation of Resistive Tearing Modes in Tokamaks

MHD STABILITY THEORY (Continued)

1A-20 A.A. Ware
Neoclassical Theory of The Mini-Disruptions

1A-21 H.V. Wong
The m=1 Resistive Kink Mode

POSTER SESSION 2A - AUXILIARY PLASMA HEATING, WAVES SIMULATION

2A-1 3:30 PM T.M. Antonsen Jr. and E. Ott
Thermal Transport in RF Lower Hybrid Heated Plasmas

2A-2 A. Bers, F.W. Chambers, N.J. Fisch and G.L. Johnston
Electron Heating in Tokamaks with RF Power at the Lower Hybrid Frequency

2A-3 V.S. Chan
The Effect of Low-Frequency Random Density Fluctuations on the Raman-Backscattering Process Near the Upper Hybrid Frequency

2A-4 C.Z. Cheng and H. Okuda
New Three-Dimensional Simulation Models for Cylindrical and Toroidal Plasmas

2A-5 S.J. Gitomer and J.L. Shohet
Computer Simulation of Radio Frequency Confinement of a Plasma Near Critical Density

2A-6 A. Hasegawa and Liu Chen
Physics of Slow Alfvén Wave Heating

2A-7 D.A. Hitchcock, S.H. Brecht and W. Horton
The Angular Dependence of Ion Cyclotron Instability In a D-T Beam-Plasma System

2A-8 G.L. Johnston, F.Y.F. Chu, C.F.F. Karney and A. Bers
Nonlinear Propagation of Lower Hybrid Waves in Two Dimensions, and Three-Dimensional OTS and Filamentation Instabilities

2A-9 J.N. Kappraff and J. Tataronis
Parametric Excitation of Alfvén and Obliquely Propagating Acoustic Waves

2A-10 J.L. Kulp, K. Theilhaber, G.L. Johnston and A. Bers
Lower Hybrid Ray Penetration into a Tokamak: Electromagnetic and 3-D Effects

2A-11 G.G. Lister, D.E. Post and R.J. Goldston
Monte Carlo Calculations of Neutral Beam Heating of Tokamaks

AUXILIARY PLASMA HEATING, WAVES SIMULATION (Continued)

2A-12 L.P. Mai and W. Horton
Anomalous Pitch Angle Scattering of Tangentially Injection Fast Ion Beams

2A-13 Y. Matsuda, J. Sampson and H. Okuda
Guiding-Center Calculation of Neutral Beam Heating for Tokamak

2A-14 B.D. McVey and J.E. Scharer
Influence of the Two-Ion Hybrid Resonance On Minority Species Heating in a Tokamak

2A-15 A.A. Mirin, M.E. Rensink and D.L. Jassby
Fokker-Planck Studies of the CIT

2A-16 G.J. Morales
The Nonlinear Coupling of Lower-Hybrid Radiation at the Plasma Edge

2A-17 C.W. Nielson, D. Winske and D.W. Hewett
A Quasi-Neutral Hybrid Model for Multi-Dimensional Plasma Simulation

2A-18 N.R. Pereira, R.N. Sudan and J. Denavit
Generation and Collapse of Two Dimensional Langmuir Solitons

2A-19 A.G. Sgro and C.W. Nielson
Hybrid Implosion Simulations of Heating in High Voltage Theta Pinches

2A-20 J.L. Shohet, S.N. Golovato and J.A. Tataronis
Analysis of a Helical Launching Structure for Alfvén Wave Heating in the Proto-Cleo Stellarator

2A-21 J.L. Sperling and C. Chu
Plasma Heating Near the Ion-Plasma Frequency

THURSDAY, 8 April

PLENARY SESSION - Chairman, D. B. Nelson

P9 9:00 AM A. Bers
RF Heating of Tokamak Plasmas

P10 9:45 AM R.L. Berger, L. Chen, C. Cheng, P.K. Kaw and F.W. Perkins
Nonlinear Processes in Lower-Hybrid Heating of Tokamak Plasmas

P11 10:10 AM H.C. Howe, J.D. Callen and L.A. Berry
High Power Neutral Beam Injection in Tokamaks

P12	10:35 AM	D.L. Jassby and R.J. Goldston Ripple-Assisted Penetration of Neutral-Beam-Injected Ions Into Tokamak Plasmas
P13		D. Fyfe, D. Montgomery and G. Joyce Turbulent Two-Dimensional Magnetohydrodynamics
P14		R.D. Hazeltine Ion Diffusion in Present Tokamak Plasmas
P15		B. Coppi and F. Pegoraro Anomalous Particle and Energy Transport in High Temperature Plasmas
P16		K. Papadopoulos, P. Palmadesso, I. Haber, N. Winsor, B. Hui and B. Coppi On the Behavior and the Consequences of Runaways and Slideaways in Tokamaks

POSTER SESSION 1B - TRAPPED-PARTICLE MODES AND TURBULANCE THEORY

1B-1	2:00 PM	C.O. Beasley, H.K. Meier, W.I. vanRij, T. Uckan and J.E. McCune Numerical Study of Drift-Kinetic Evolution of Collisional Plasmas in Tori
1B-2		P.J. Catto, K.T. Tsang and J.D. Callen Collisional Effects on Trapped Electron Instabilities
1B-3		K.R. Chu, W. Hobbs, W.M. Manheimer and W.M. Tang Numerical Study of the Dissipative Trapped Electron Mode in Tokamaks
1B-4		M. Ciarvella and R.E. Aamodt Nonlinear Ion Motion in a Slow Ion Cyclotron Mode
1B-5		B.I. Cohen, J.A. Krommes, W.M. Tang and M.N. Rosenbluth Nonlinear Saturation of the Dissipative Trapped-Ion Mode Coupling
1B-6		J.D. Gaffey and C.S. Liu Trapped-Electron Mode Driven Unstable by Neutral Beam Injection in Tokamaks
1B-7		S. Hamasaki, N.A. Krall and C.E. Wagner Modeling of Turbulent Theta Pinches With a Hybrid Code
1B-8		F.L. Hinton and T.B. Moore Impurity Acoustic Instability in the Trapped Electron Regime
1B-9		W. Horton, Jr. Spectral Distribution of TE-Mode Fluctuations Predicted by a 3D Mode Coupling Analysis

TRAPPED-PARTICLE MODES AND TURBULANCE THEORY (Continued)

1B-10		R.A. Koch, D. Choi and W. Horton Scaling Laws for Ion Acoustic Turbulence
1B-11		Y.Y. Kuo, W.W. Lee and H. Okuda Theory of the Nonlinear Saturation and Anomalous Transport due to Drift Instabilities
1B-12		H.C. Lui and T.C. Marshall Numerical Simulation of a Turbulent Heated Plasma for A "High Beta Tokamak"
1B-13		W.M. Manheimer, K.R. Chu, E. Ott and J.P. Boris Marginal Stability Calculation of Trapped Electron Mode Anomalous Transport in Tokamaks
1B-14		F. Pegoraro and B. Coppi Fluid-Like Instabilities Driven by the Magnetic Curvature Drift
1B-15		G. Rewoldt, W.M. Tang and E.A. Frieman Two-Dimensional Spatial Structure of the Dissipative Trapped-Electron Mode
1B-16		A. Simon Small Amplitude Nonlinear Saturation of Finite, Collisionless Plasma
1B-17		G.R. Smith and A.N. Kaufman Particle Heating by Diffusion in a Single Wave
1B-18		W.M. Tang, J.C. Adam and D.W. Ross Residual and Ion Diamagnetic Trapped-Particle Modes
1B-19		N.K. Winsor and B.H. Hui Numerical Studies of Neoclassical Resistivity in Tokamaks

POSTER SESSION 2B - EXPLORATORY CONCEPTS AND GENERAL PLASMA THEORY

2B-1	3:30 PM	J. Allbritton, R. Cohen, R.S. Devoto and G. Rowlands Flute Instability of a Laser-Pellet Plasma in a Magnetic Mirror
2B-2		K. Audenaerde Cyclotron Radiation in Fusion Experiments
2B-3		D-I. Choi and W. Horton Stability of the High Energy Runaway Electron Component in a Magnetized Plasma
2B-4		G. Conn, P. Rosenau, J. Tataronis and H. Weitzner On the Effects of a Hall Current in Magnetohydrodynamics

EXPLORATORY CONCEPTS AND GENERAL PLASMA THEORY (Continued)

FRIDAY, 9 April

PLENARY SESSION - Chairman, G. E. Guest

POSTER SESSION 1C - TRANSPORT PROCESSES

TRANSPORT PROCESSES (Continued)

TRANSPORT PROCESSES (Continued)

POSTER SESSION 2C - HIGH-BETA, PINCHES, PLASMA SIMULATION, AND BOUNDARY LAYER THEORY

HIGH-BETA, PINCHES, PLASMA SIMULATION, AND BOUNDARY LAYER THEORY (Continued)

QUASILINEAR CALCULATIONS OF PLASMAS IN MIRROR MACHINES*

H. L. Berk

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Livermore, California 94550

ABSTRACT

In order to describe the nonlinear consequences of loss cone instabilities (in particular the drift cyclotron loss cone mode) together with classical processes in mirror machines, we have incorporated quasilinear diffusion models in one and two-dimensional velocity space transport codes. The one dimensional model, which describes the perpendicular velocity distribution, is applicable when the parallel ion temperature is sufficiently small. The resulting plasma parameters such as electron temperature, density, and energy containment time correlate closely with results obtained in the 2XIIB experiment when a plasma stream is present. For mirror machines with larger radial scale lengths the results become more sensitive to the detailed dispersion relation and computer experiments are currently being run in this regime in order to optimize containment with respect to external stream.

In addition the Hybrid II Fokker-Planck code has been altered to incorporate the quasilinear diffusion model. For mirror machines, where the radial scale length is only a few Larmor radii, this code reproduces many of the features of the one-dimensional code. In particular it demonstrates that the ion-ion scattering does not strongly affect the evolution of the system. The most important modifications of the results from the one dimensional code was due to more realistic descriptions of electron transport and the transit loss of ions in the loss cone.

*Work performed under the auspices of the U.S. Energy Research and Development Administration under contract No. W-7405-Eng-48.

SIMULATION OF HIGH β PLASMAS IN MIRROR MACHINES*

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ABSTRACT

The nonlinear particle code Superlayer has been used to simulate high β plasmas in 2XIIB; it consistently predicts steady state values of $\beta > 1.0$, also recently achieved in 2XIIB. This result is in conflict with theoretical predictions of limits on $\beta \approx .1-.2$, due to the alfven ion cyclotron (AIC) mode. The code should be subject to modes of the mirror-tearing mode type and also to the AIC mode (although only in a modified form due to code restrictions). The studies to date have concentrated on large orbit systems, r_p/ρ_i , $L/\rho_i \lesssim 2$. The few runs that we have attempted for much larger systems indicate that they will either tear apart or shrink to smaller size. Precise limits on lengths have not been obtained.

We are also developing linearized particle codes, that start from a steady state equilibrium, either forced or self consistently obtained from Superlayer, and follow the development of linearized displacements from the equilibrium state. Initial results here also indicate that small size is favorable to higher β. In one model the AIC mode is stable at $\beta = 1$ for $T_\perp/T_\parallel = 5$ if $\frac{L}{\rho_i} \approx 1.$, whereas the allowable $\frac{T_\perp}{T_\parallel} \lesssim 2$ for longer systems. These linearized codes will allow us to examine modes of the form $e^{im\theta}$, $m \neq 0$.

*Work performed under the auspices of the U.S. Energy Research and Development Administration under contract No. W-7405-Eng-48.

LOW-FREQUENCY STABILITY OF FIELD REVERSED ION RINGS*

R.N. Sudan, Cornell University, & M.N. Rosenbluth, Institute for Advanced Study

The low frequency stability of a field reversed ion ring in a plasma background has been treated by developing a sufficient "energy principle" which takes into account the effects of the large Larmor orbits of the ring ions. The calculation begins by recognizing the system free energy

$$C = U - \Omega L_z - T\int d^3x d^3v f \ln f$$

where U is the total system kinetic and electromagnetic energy, L_z is the z-component of the total angular momentum, f is the ring ion distribution and T its temperature. In equilibrium f_o is chosen to be a monotonically decreasing function of $H-\Omega P_\theta$ where H and P_θ are the particle energy and canonical angular momentum respectively and Ω is mean angular velocity. The collisionless fluid equations govern the plasma motion (with neglect of electron inertia) while the ring ion dynamics is determined by the Vlasov equation. The first order perpendicular displacement of the electron fluid is denoted by $\underline{\xi}_\perp$. The change in C vanishes to first order in $\underline{\xi}_\perp$ because of the appropriate equilibrium conditions but to second order $\underline{\xi}_\perp$ we obtain for $\omega < \Omega$, $\delta C = \delta W_{MHD} + \delta W_c + \delta W_b$

where
$$\delta W_{MHD} = \int \frac{d^3x}{4} \{|\underline{B}^{(1)}|^2 - \underline{\xi}_\perp \cdot \underline{J} x \underline{B}^{(1)} + \underline{\xi} \cdot \underline{J} x \underline{B} (\nabla \cdot \underline{\xi})\}$$

$$\delta W_c = \int d^3x \, \underline{\xi} \cdot \underline{J} x \underline{B} (mr\Omega^2/T)(\xi_\perp \cdot \hat{r})$$

$$\delta W_b = \frac{e^2}{T} \int d^3x d^3v f_o |g|^2$$

where $g = i(\omega - \ell\Omega)\int_{-\infty}^{t} dt' \underline{\xi}' \cdot \underline{v}' x \underline{B}$, and $\underline{B}^{(1)} = \nabla x \underline{\xi} x \underline{B}$. Here, $\underline{J}$ and $\underline{B}$ are the ring current and total magnetic field, g is an orbit integral, ℓ is the azimuthal mode number of the perturbation. Stability is assumed for $\delta C > 0$; δW_{MHD} is the conventional magnetohydrodynamic δW, δW_c is the destabilizing influence of the centrifugal force and δW_b is a positive definite stabilizing term from finite Larmor orbit effects. In the case of thin infinitely long layer we obtain that $\delta C > 0$ for all modes $\ell \geq 2$ for $\ell = 1$, $\delta C > 0$ for $k^2R^2 > 0.15$ and for $\ell = 0$ we recover the conventional tearing instability; here k is the perturbation wave number in the z-direction and R is the mean layer radius.

*Work supported by ERDA and Naval Research Laboratory.

Diffusive Transition from Belt Pinch to Doublet

D.C. Stevens, E. Turkel
New York University

The fact that the classical skin effect in a plasma is not the same as in a solid conductor (e.g. it does not exist at all in a bumpy torus) and that, in a Tokamak, it couples and interferes with classical (Pfirsch-Schluter) and Neoclassical plasma diffusion was pointed out by Grad and Hogan in 1969. For numerical purposes, the crucial simplification[1] is to separate out one-dimensional diffusion between flux surfaces (or in the adiabatic case, an average pressure balance) from the occasional two-dimensional calculation of geometrical (inductance) coefficients in the diffusion equation. We present results of an efficient numerical 2-D diffusive transition from a Belt Pinch to a Doublet configuration with combined toroidal, poloidal and shaping field buildup. Finite resistivity changes the singularity at the separatrix from the one found in an adiabatic compression.[2] The diffusive code is equally efficient for large resistivity (approximately uniform plasma current) and for small resistivity approximating the adiabatic results but with explicit boundary layers near the separatrix and plasma edge. The Pfirsch-Schluter and Grad-Hogan diffusion velocity profiles are compared.

1 H. Grad, Oak Ridge Memoranda, April 22, 1970 and July 6, 1970.

2 H. Grad, P.N. Hu, D.C. Stevens, Proc. Nat. Acad. Sci., 72, 3789 (1975).

<u>Axisymmetric Stability in Tokamaks</u>.* M. S. Chu, R. L. Miller, and T. H. Jensen, General Atomic Company--The energy principle is used to investigate numerically the stability of arbitrary cross-section tokamak equilibria to axisymmetric modes. Two-dimensional trial functions are used to calculate the plasma contribution to the energy integral while the vacuum contribution is evaluated using Green's function techniques. The modes considered are compressible and the mode structure can be interpreted as actual motion of the plasma when the inertia of the plasma in the flux surfaces is neglected. Stability results for ellipses and doublets will be shown for various current profiles and aspect ratios. Toroidal effects are isolated by comparing results to the limiting case of a straight system subject to incompressible perturbations. Stabilization by a conducting wall is always possible, provided the wall is sufficiently close to the plasma surface.

*Work supported by the U. S. Energy Research and Development Administration, Contract E(04-3)-167, Project Agreement No. 38.

Saturation of the Tearing Mode*

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Princeton University, Princeton New Jersey

M. N. Rosenbluth and B. V. Waddell
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ABSTRACT

The growth and saturation of magnetic islands in tokamaks is important because of their possible large influence on radial transport. Once the island width is greater than the tearing layer width, the growth of the island is governed by the equation $\left\langle \frac{\partial\psi}{\partial t} \right\rangle = \langle \eta \rangle \; J(\psi)$ where $J(\psi)$ is the current, ψ the helical flux, η the resistivity, and the brackets indicate averages over flux surfaces. As shown by Rutherford,[1] exponential growth ceases at this point and the island width grows linearly in time. We have constructed an analytical model of the saturation process which is a quasilinear extension of the theory of Rutherford. The results of the model are in agreement with those obtained by time stepping a numerical code described previously. The saturation of the tearing mode is the quasiliner development of a single mode rather than a mode coupling process. The amplitude is dependent on the form of the resistivity. Applications of the model will be discussed.

[1]P. H. Rutherford, Phys. Fluids <u>16</u>, 1903 (1973).

*Research sponsored by the U.S. Energy Research and Development Administration, Grants No. E(11-1)3237 and E(11-1)3073.

Tokamak Sawtooth Oscillations Due to Tearing Modes*

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Abstract

Sawtooth oscillations are a striking feature of the soft x-ray emission from tokamaks. It is shown that sawtooth oscillations can be obtained from the quasilinear theory of heat transport due to tearing modes [1], in particular the drift-tearing mode found by Hazeltine, Dobrott, and Wang[2].

The theory can account for many of the experimental observations, including: (1) the sawtooth oscillations, (2) the sawtooth period, (3) the localization of the effect to mode rational radii r_s at which $q(r_s) = m$, (4) the lack of mode structure of the sawtooth, (5) the sawtooth amplitude, (6) the presence of smaller-amplitude "precursor" oscillations with frequency ω_*, the electron diamagnetic frequency, (7) the loss of energy confinement, (8) the inversion of the sawtooth waveform across r_s.

The presence of the tearing mode causes a localized anomalous thermal conductivity κ_*, which is proportional to the amplitude squared of the mode. The growth rate of the mode is a sensitive function of the temperature gradient at r_s. Oscillations of the temperature are found with frequency $\approx \omega_* (\tau \kappa_\perp / n)^{\frac{1}{2}} / \lambda$, which can be identified with the sawtooth frequency. (Here $\kappa_\perp$ is the background thermal conductivity, λ is the tearing layer width, n is the density, and τ is the collision time). Numerical results show that the nonlinear oscillations have the characteristic sawtooth waveform.

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1. H. R. Strauss and R.V. Hazeltine, Bull. Am. Phys. Soc, Oct. 1974.
2. R.D. Hazeltine, D. Dobrott, and T.S. Wang, Phys. Fluids 18, 1778, (1975).

*This work was supported by U.S. ERDA Contract E-(40-1)-4478.

ABSTRACT (To be presented at the CTR Sherwood Meeting of April 1976). "Equilibrium and Stability Code for a Diffuse Plasma in Three Dimensions". Octavio Betancourt and Paul Garabedian. ERDA Mathematics and Computing Laboratory, New York University, New York, N.Y. 10012

A computer code to investigate the equilibrium and stability of a diffuse plasma in three dimensions is described that generalizes earlier work on a sharp free boundary model. Toroidal equilibria of a plasma are determined by minimizing the potential energy of the system, subject to appropriate constraints. This is accomplished by considering accelerated paths of steepest descent associated with a nonlinear version of the variational principle of magnetohydrodynamics that involves mapping a fixed coordinate domain onto the plasma. Discrete equations are derived by using the finite element, and an artificial time iteration is suggested by the conjugate gradient method. Stability is tested by checking whether the energy has a relative minimum.

As an application of the code, diffuse equilibria for SCYLLAC are determined. Also, it is found that certain internal kink modes are stabilized by the effect of curvature in a torus of low aspect ratio.

This work was supported by the U.S. Energy Research and Development Administration under contract E(11-1)-3077 with N.Y.U.

P9

Review Paper

RF HEATING OF TOKAMAK PLASMAS

Abraham Bers
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

P10

Nonlinear Processes in Lower-Hybrid Heating of Tokamak Plasmas*

R.L. Berger, L. Chen, C. Cheng, P.K. Kaw, F.W. Perkins
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Nonlinear processes play a crucial role in the attempts to heat tokamak plasmas by radiation near the lower-hybrid frequency. We assume, as has been the case experimentally, that energy propagates into tokamaks via electrostatic waves launched by phased waveguide arrays. The waves propagate in well-defined resonance cones. We show that, at the power levels sufficient to heat tokamaks, convective decay instabilities often break out in the resonance cones and nonlinear saturation calculations show that severe surface anomalous absorption occurs. The pump depletion associated with the anomalous absorption prevents the occurance of filamentation instabilities. Theoretical predictions of the threshold power and decay spectrum agree well with experiments. Our present understanding is that effective heating of tokamaks can be carried out only by avoiding the nonlinear regimes or perhaps by initially launching whistler mode waves.

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HIGH POWER NEUTRAL BEAM INJECTION IN TOKAMAKS*

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The behavior of a next generation tokamak heated primarily by neutral beam injection is studied with a one-dimensional transport model which includes a detailed, classical treatment of fast ion deposition and slowing down. Comparison of the model with recent confinement and heating experiments in ORMAK shows that 1) the ion temperature obeys neoclassical scaling, 2) the electron temperature is well described by an empirical heat conduction scaling $\chi_e \alpha I_T/n_e^2$, and 3) the observed ion temperature increase due to neutral beam heating (up to P_{inj} = 360 kW) agrees within experimental uncertainties with classical predictions. The large density increase obtained with gas puffing is not understood, and the plasma density is therefore taken as an independent variable in these studies.

For studies of the effects of increased neutral injection power, nominal values of machine size and toroidal current and field characteristic of next generation tokamaks are assumed, and the plasma density is taken to be as high ($\sim 10^{14}$ cm^{-3}) as can be penetrated by the neutral beam in the presence of impurities. Several megawatts of injected power raise the ion temperature to several keV; the ohmic heating is almost negligible and the electrons are collisionally heated by the ions. Beta poloidal (β_p) values in excess of the aspect ratio may be obtained by lowering the toroidal current while maintaining the plasma pressure with beam heating. This test of high-beta equilibrium is not inhibited by either decreased plasma energy confinement or trapped fast ion loss to the wall (loss cones) at the lower currents. Enhancement of ion heat conduction losses by toroidal field ripples is small even as the ions approach the collisionless regime. Toroidal rotation of the plasma due to beam-imparted momentum may be large and is limited primarily by field ripple. By lowering the plasma density to allow the beam to stack, the ratio (Γ) of the energy in the fast ions to the energy in the plasma may be raised to $\Gamma \sim 1$ while the toroidal current in the beam may approach the total toroidal current. These experiments will extend the operational range of tokamaks into temperature, β_p and Γ regimes relevant to the EPR.

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RIPPLE-ASSISTED PENETRATION OF NEUTRAL-BEAM-INJECTED IONS INTO TOKAMAK PLASMAS*

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The penetration length of energetic neutral beams injected into a tokamak plasma can be effectively increased by a large factor, if the beams are injected vertically into a region where the toroidal-field ripple, $\varepsilon(z)$, has a top-bottom asymmetry. For injection from the bottom, a large ripple is created below the horizontal midplane ($z = 0$), while the ripple is made to fall very rapidly above the midplane, so that at $z > 0$ there exists a substantial region where ripple magnetic wells are not possible at low q. Energetic ions formed from injected neutral beams are mirror-trapped in the ripple magnetic well, and drift upward with the $\vec{B} \times \nabla B$ velocity, v_d, to the midplane region, where they are captured. An ion can reach the midplane region without scattering into a banana orbit at $z < 0$, if $a/v_d \gtrsim \varepsilon_o \tau_{sc}$, where ε_o is the peak-to-peak ripple at $z \approx 0$, $\varepsilon(z < 0) > \varepsilon_o$, and τ_{sc} is the 90° scattering time. When $\varepsilon_o \sim 0.01$, 110 to 150-keV deuterons can be deposited near the center of plasmas with $\langle n_e Z_{eff} \rangle a \sim 10^{17}$ cm^{-2}. The conventional injection method requires an order-of-magnitude larger beam energy.

The feasibility of the ripple-assisted penetration technique has been demonstrated by orbit calculations with a Monte Carlo guiding-center code. Ions with energy in the range just sufficient to reach the midplane are captured in the central plasma region, and are confined until they thermalize.

Associated problems now under investigation are the loss of very energetic ions, such as fusion alpha particles, by a ripple-driven orbit diffusion process, and the possibility of convective-loss-cone instability set up by the highly peaked fast-ion velocity distribution in the ∇B-drift region.

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TURBULENT TWO-DIMENSIONAL MAGNETOHYDRODYNAMICS

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The formation of internally-generated magnetic fields of macroscopic dimensions is a common (and ill-understood) phenomenon in many plasma situations: megagauss magnetic fields in laser-irradiated pellets, magnetic dynamo generation in the earth's interior, formation of "magnetic islands" in tokamaks, consolidation of electric current filaments in relativistic electron-beam simulations. Explanations in terms of linear instabilities and eigenmodes do not appear to be adequate. We propose an alternative mode of description in terms of fluid turbulence theory, generalized to MHD situations. Turbulent energy cascades to shorter wavelengths are familiar from Kolmogoroff theory in 3D Navier-Stokes fluids. Less familiar but now well established for 2D Navier-Stokes fluids are the simultaneous cascades of enstrophy to shorter wavelengths and energy to longer wavelengths. The macroscopic vortex structures which result are of importance for meteorology on planetary scales and for "convection-cell" formation in electrostatic guiding center plasmas.[1] Two-dimensional MHD has three inviscid constants of the motion[2] which survive a truncation in Fourier space and which, in the presence of dissipation, can be cascaded in strongly-turbulent situations: energy E, cross-helicity P, and mean-square vector potential A. It appears likely, using arguments adapted from Navier-Stokes theory, that in small-helicity situations, energy will cascade to shorter wavelengths and vector potential to longer ones. The result of such cascades would be an accumulation of magnetic excitation at the longest wavelengths in the system, forming "magnetic islands", analogous to the macroscopic electrostatic vortex structures of the low-beta situation. Inviscid equilibrium solutions are equipartition spectra for magnetic field energy and kinetic energy at short wavelengths, but sharply peaked magnetic spectra at long wavelengths. An implication for fusion devices is that two-dimensional geometries, where symmetry considerations indicate independence of one spatial coordinate, may be more susceptible to disruptive behavior than three-dimensional ones. Numerical simulations have been performed for the inviscid MHD equations using the spectral method of Orszag for different initial values of E, P, and A. Results for the evolving spectra are compared with theoretical preditions.

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[2] D. Fyfe and D. Montgomery, University of Iowa Report 75-44, "High-Beta Turbulence in Two-Dimensional Magnetohydrodynamics" (submitted for publication).

Ion Diffusion in Present Tokamak Plasmas

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Poloidal variation of the electrostatic potential in an impure tokamak plasma can be substantial, and it significantly alters the nature of neoclassical diffusion. A previous study of such diffusion[1] is here made complete, through calculation of the Pfirsch-Schlüter contribution to the hydrogen ion flux. Because the hydrogen ions are assumed to be in the plateau regime of neoclassical transport theory, this contribution is smaller than the more familiar plateau-type particle flux; it nonetheless determines the relative directions of hydrogen and impurity ion diffusion. Calculation of the Pfirsch-Schlüter flux in the presence of a poloidal electrostatic field requires significant extension of neoclassical kinetic theory: the strong electrostatic coupling to low energy ions leads to a logarithmic term in the lowest order diffusion coefficient. The present analysis yields improved agreement with tokamak experimental observations, especially in that a tendency for impurity ions to concentrate near the magnetic axis is no longer predicted.

[1] R. D. Hazeltine and A. A. Ware, to be published in The Physics of Fluids.

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Anomalous Particle and Energy Transport in High Temperature Plasmas*

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Among the possible models that can be invoked to explain the improvement of energy replacement time when the density is increased, as observed in various toroidal confinement experiments and in Alcator in particular, we have developed one involving the excitation of current driven electron drift modes.

We consider regimes, as found in practice, where the current carrying electron distribution is affected by the existence of a relatively small region in velocity space of trapped electrons. Then an applied electric field, considerably smaller than the runaway critical field, can produce a sufficiently positive slope of the electron distribution in the direction of acceleration to excite the above mentioned modes.

An estimate of the relevant diffusion coefficient, with the assumption that the fluctuation amplitude $\tilde{\Phi}$ at which the instability saturates is such that $k_\perp c\,\tilde{\Phi}/B \approx |d \ln n/dr| cT_e/(eB)$, leads approximately to a scaling of the energy replacement time $\tau_E \propto n\, q_0 \sqrt{T_e}$, where $q_0 \propto B/I$, and of the electron temperature $T_e \propto I\sqrt{q_0}$. Here B is the magnetic field, I the plasma current, $k_\perp$ the transverse wave number and q_0 the stability safety factor.

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ON THE BEHAVIOR AND THE CONSEQUENCES OF RUNAWAYS AND SLIDEAWAYS IN TOKAMAKS

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A study of the behavior of runaways and slideaways in tokamaks is presented with emphasis on the interpretation of the recent Alcator, TM-3 and T-6 results. The study is a combination of analytic and numerical techniques, including use of Fokker-Planck and particle type codes.

It is shown by particle simulations that the presence of a positive slope in the distribution function of either the slideaways or the runaways can excite lower hybrid waves and can produce a substantial amount of ion heating in a time sequence that corresponds to the experimental results. In addition the nonlinearly stabilized spectrum has most of its energy in the frequency range near the lower hybrid. A two dimensional mode coupling scheme is presented which shows that nonlinear Landau damping tends to rotate the spectrum in the perpendicular direction so that it collapses to the minimum allowable frequency (Ω_{LH}).

For the slideaway case the positive slope appears in the collisional Fokker Planck type simulations due to the presence of trapped electrons. For the runaway case, it is shown that for $\frac{\Omega_e}{\omega_e} \gg 1$, a positive slope develops collisionlessly. This is due to the fact that a kinetic cyclotron instability independent of the slope of the runaways selectively scatters particles with parallel velocities $v_\| > \frac{\Omega_e}{\omega_e} V_e$. In the presence of an electric field, the hole in the diffusion coefficient for velocities in the range $V_e < v_\| < \frac{\Omega_e}{\omega_e} V_e$, piles-up particles near $v \approx \frac{\Omega_e}{\omega_e} V_e$, and leads to a positive slope. This effect is studied analytically and by particle and Fokker-Planck type simulations.

Finally the overall picture emerging from the competition of the quasilinear flattening of the bump due to the Cerenkov instability and the formation of the bump due to the cyclotron instability is presented. Several scalings and important consequences with respect to cyclotron and lower frequency radiation from high and low density tokamaks will be discussed.

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†Science Applications Inc.
*Massachusetts Institute of Technology

EQUILIBRIUM AND STABILITY STUDIES OF A HIGH BETA TOKAMAK*

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Both equilibrium and stability studies have been made to investigate the upper limits on β in a tokamak. The equilibrium studies are carried out by solving the well known flux equation

$$\Delta^*\psi = -[\mu_o R^2 p' + II']$$

A variety of profiles are used and the maximum β's possible are calculated for a fixed aspect ratio and q = 1 on axis. For profiles in which the current falls off sufficiently rapidly at the edge of the plasma (to avoid high m number instabilities), high peak β's are possible, $\beta \sim$ 10-20%. However, because the profiles are narrow the average β is only of the order $\beta \lesssim 2\%$, even when a/R = .4. Higher β's are possible with wider profiles but can only be justified by assuming that higher m modes are not very dangerous or else are stabilized by the presence of a conducting wall quite close to the plasma edge. The highest β values are achieved by inducing substantial amounts of force free currents outside the main bulk of plasma, as originally proposed by the Dutch group in the SPICA experiments.

A new stability calculation has also been carried out to determine the influence of force free currents on the stability of global kink modes. The model studied is a surface current model with constant plasma pressure. This plasma is surrounded by a force free magnetic field, satisfying $\nabla \times \underline{B} = \alpha\underline{B}$, α = const. The calculation is carried out using the high β tokamak expansion and it is found that by optimally choosing α, the critical β is increased by a factor of about three, over the corresponding case with vacuum on the outside.

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FLUX CONSERVING HIGH BETA TOROIDAL EQUILIBRIA*

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Successful auxiliary heating requires τ(heating) $\lesssim \tau_E < \tau$ (magnetic diffusion). The ensuing constraints of flux conservation for neighboring equilibria with increasing pressure leads to certain radical departures from standard low beta tokamak scaling laws. Extending the virial theorem and the integral form of the force balance of ideal MHD from the quasi cylindrical low beta case given by Shafranov[1] to high $\beta \sim \varepsilon$ toroidal equilibria, we arrive at the following conclusions:

1) q remains constant.
2) the equilibrium limit given by a second magnetic axis entering the plasma is removed.
3) the characteristic tendency of flux conserving equilibria toward formation of surface currents can be relaxed by allowing the plasma to expand in minor radius, retaining a net gain in β.

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[1]V. D. Shafranov, Reviews of Plasma Physics 2, 106.

P19

Drift Wave Turbulence and Shear Stabilization in the Presence of Multiple Rational Surfaces*

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Two important aspects of the collisionless drift wave instabilities have been studied numerically using a recently developed particle simulation code[1]. They are (1) the nonlinear behavior of the instability and the associated plasma transport in a system where many linearly unstable modes are simultaneously present, and (2) the effect of the shear on the stabilization of the drift instability in the presence of multiple rational surfaces. These situations are expected to exist in large confinement devices such as tokamaks.

We have shown previously, for a system with only a single unstable mode, the nonlinear saturation is due to the quasilinear density decay and the anomalous heat transfer is caused by the Landau absorption of the wave energy[2,3]. When there are many unstable modes present, our results indicate that these observations remain valid and that the instability is dominated by a few linearly most unstable modes which are responsible for the anomalous plasma transport. Effect of the mode coupling appears small, probably due to the dispersive nature of the drift instability.

We have also shown by simulation[2] that the conventional Pearlstein-Berk criterion for the shear stabilization is valid for the case where there is only one rational surface, namely, the normal modes are localized on the well-separated surfaces. However, if the distance between the rational surfaces is close enough to cause the overlaping of the neighboring modes, the global mode structure will then be developed bridging over different mode rational surfaces. Therefore the stabilization criterion may be strongly modified. This was investigated analytically and by numerical simulation. Our results indicate that the overlaping of the localized modes reduces considerably the effectiveness of shear stabilization. Details will be reported.

(1) W.W. Lee and H. Okuda, Matt-1231.
(2) W.W. Lee and H. Okuda, Matt-1188.
(3) Y.Y. Kuo, W.W. Lee, and H. Okuda, this conference.

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P20

High Beta MHD Stability and Bifurcation for High Order Multipole and Surface Magnetic Field Configurations*

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Problems of high beta equilibrium and stability for high order multipole or surface magnetic field confinement configurations differ somewhat from those for more conventional configurations because the magnetic field is very small over a large region of the plasma volume. For high order multipoles, the surface magnetic field layer can be well approximated by a plane slab and the free boundary problem can be analytically solved, in closed form, using conformal mapping techniques. Three configurations were examined in detail: the simple magnetic slab (or thicket fence), consisting of conductors carrying current $\pm I$ at $x = nd$, $y = \pm a/2$ $(n=0,1,2,\ldots)$; the staggered slab, in which one set of conductors is displaced in x by d/2; and the familiar picket fence, the $a = 0$ limit of the staggered slab. In each case, the dimensionless parameters $\bar{a} = \beta_a^{1/2} = a/w$ and $\bar{d} = \beta_d^{1/2} = d/w$, with $w = 2I/c(8\pi p)^{1/2}$, p = scalar plasma pressure, completely characterize the problem. We find that the region of MHD stability in the $\bar{d}$, $\bar{a}$ plane is much larger for the staggered slab than for the simple slab. In both cases, we find bifurcation: there exists a region of the $\bar{d}$, $\bar{a}$ plane in which, for each set of $\bar{d}$, $\bar{a}$ values, there are two distinct solutions, one stable and one unstable. In addition, there are regions of the $\bar{d}$, $\bar{a}$ plane in which there are only stable solutions; only unstable solutions; or no solutions at all of the topology considered here (i.e., single, continuous free boundary separating plasma in one half space from magnetic field and vacuum in the other). Of course, solutions of different topological character may also exist (e.g. isolated, cylindrical bubbles of vacuum and magnetic field around each conductor or each pair of conductors.) Although present experiments are not at sufficiently high beta values to test these calculations, a Marshall gun directed at a plane or cylindrical slab might provide confirmation of the calculated stability limits as well as an indication of the diffusion rate through the slab.

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THEORETICAL INTERPRETATION OF STABILITY AND TRANSPORT IN EBT*

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In the ELMO Bumpy Torus (EBT) experiment an interesting regime of operation has been developed which appears to be macroscopically stable. The parameters for this regime are in reasonable agreement with a simple neoclassical point model of scaling and transport.

Relevant stability calculations for EBT include both Vlasov and MHD or Guiding Center analyses, which we briefly review and interpret.

The simple neoclassical point model employed yields explicit solutions for steady state. The thermal stability boundaries are also particularly simple and correspond to relative extrema in the neutral density. The model, while optimistic, is encouraging in that it predicts a regime of operation in which the density, temperature and lifetime all increase simultaneously. The presentation will emphasize comparisons with present experiment and implications for future experiments in the EBT sequence. Interrelation with further developments in the theory, discussed elsewhere in this meeting, will also be discussed.

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INFLUENCE OF FINITE-BETA TRANSVERSE ELECTROMAGNETIC EFFECTS ON CROSS-FIELD INSTABILITIES IN HIGH-DENSITY PINCHES*

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Existing linear and nonlinear theories of current-driven microinstabilities in high-density pinches are based on the electrostatic approximation. In this paper we investigate the influence of finite-beta transverse electromagnetic effects on the lower-hybrid-drift instability, which has recently received considerable attention as a mechanism for post-implosion anomalous transport [1], flute perturbations during implosion [2], as well as an anomalous transport mechanism during the implosion [3]. Linear stability properties of the lower-hybrid-drift instability are studied assuming flute-like perturbations with $\underset{\sim}{k}\cdot\underset{\sim}{B}_0=0$, $\delta\underset{\sim}{B} \parallel \underset{\sim}{B}_0$ and $\delta\underset{\sim}{E} \perp \underset{\sim}{B}_0$. The analysis is carried out in slab geometry with $\underset{\sim}{k}=k_y\hat{\underset{\sim}{e}}_y$ (the cross-field current direction), and inhomogeneities in density, temperature and magnetic field in the x-direction. The analysis, valid for arbitrary T_e/T_i and V_E/v_i (here V_E is the cross-field electron $\underset{\sim}{E}\times\underset{\sim}{B}$ velocity, and v_i is the ion thermal speed), includes ∇B resonance effects and leads to a transcendental integral equation for the complex eigenfrequency $\omega + i\gamma$. This equation is solved numerically, and the parametric dependence of maximum growth rate on T_e/T_i, β_i, and V_E/v_i is discussed. The general tendency is for finite-beta effects to reduce the growth rate relative to previous electrostatic analyses. For example, in the limiting case where $T_i \gg T_e$ and $V_E/v_i \lesssim 1$, the maximum growth rate (which can be calculated analytically in this regime) is given by $(\gamma_M)_{EM}=(\gamma_M)_{ES}/(1+\beta_i/2)^{1/2}$, where $\beta_i=8\pi nT_i/B_0^2$ is the local ion beta, and $(\gamma_M)_{ES}=(2\pi)^{1/2}(V_E/v_i)^2\Omega_{LH}/8$ is the maximum growth rate based on an electrostatic stability analysis [1]. On the other hand, for finite values of T_i/T_e, the lower-hybrid-drift instability is completely stabilized by electromagnetic effects above some critical value of beta. For example, for $V_E/v_i = 1$, $\omega_{pe}^2/\omega_{ce}^2 \gg 1$ and $T_i/T_e=1$ instability ceases whenever the local beta satisfies $\beta_e+\beta_i>\beta_{cr}=1.6$.

*Research supported by Energy Research and Development Administration.
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[3] P. C. Liewer and N. A. Krall, Phys. Fluids 16, 1953 (1973).

P23

FINITE-GYRORADIUS STABILIZATION OF INSTABILITIES IN SHARP-BOUNDARY SCREW PINCHES*

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Freidberg[1] proposed a model for studying finite-ion-gyroradius effects in high-beta pinches, the Vlasov-fluid model, which is appropriate for the treatment of low frequency phenomena ($\omega \ll \omega_{pi}$). Ions are treated as collisionless. The electrons are treated as a cold, massless fluid. Charge neutrality is assumed and the displacement current is ignored.

We are using two methods of elucidating the linear stability theory of the Vlasov-fluid sharp-boundary screw pinch, one computational[2] and the other analytical.[3]

Our numerical approach yields the eigenfrequencies of the normal modes of the perturbed screw pinch. These eigenfrequencies are approximated by the roots of a polynomial.

For the set of parameters used, we find negligible effect upon the m = 1 instability due to finite ion temperature, whereas the m = 2 instability is stabilized for sufficiently large temperature, in qualitative agreement with the results of Freidberg.[1] However, the critical value of the ratio of ion gyroradius to plasma radius above which the m = 2 mode is stabilized is <u>lower</u> than was predicted by Freidberg by a factor of about 2.5. This factor agrees well with the data obtained from recent experiments at Garching and Los Alamos, as well as with the recent analytical treatment of Turner.[3] The smaller critical ratio enhances the possibility of simultaneous wall stabilization of the m = 1 instability and finite-gyroradius stabilization of the m = 2 instability in near-theta-pinch machines such as Scyllac.

1. J. P. Freidberg, Phys. Fluids <u>15</u>, 1102 (1972).

2. H. Ralph Lewis and Leaf Turner, "Stability Analysis of Sharp-Boundary Vlasov-Fluid Screw-Pinch Equilibria," Third Topical Conference on Pulsed High-Beta Plasmas, Culham Laboratories, England, September 9-12, 1975.

3. Leaf Turner, "The Sharp-Boundary Screw Pinch; Kinetic Corrections to Magnetohydrodynamics According to Vlasov-fluid Theory," submitted to this conference; Leaf Turner, "Finite-Larmor-Radius Stabilization in a Sharp-Boundary Vlasov-Fluid Screw-Pinch," Los Alamos Scientific Laboratory report LA-UR-76-397, submitted for publication.

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P24

A COMPUTATIONAL STUDY OF THE POST-IMPLOSION DYNAMICS OF THE SCYLLAC EXPERIMENT*

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The question of equilibrium of a diffuse plasma in the Scyllac experiment is addressed using a numerical algorithm for the solution of the nonlinear equations describing ideal, magnetohydrodynamic flow in three dimensions. This time dependent approach illuminates both the transient response of the plasma and the approach to toroidal force balance.

The numerical algorithm employs a computation mesh whose boundary conforms to the shaped compression coil. The mesh is divided by a Lagrangian interface separating the dense, imploded plasma column and the surrounding vacuum. Within the plasma region, the equations for magnetohydrodynamic flow are solved using the method described by Brackbill. Exterior to this region, the vacuum field equations are solved using a recently developed block iterative, successive over-relaxation method. A constant toroidal flux is specified on the outer boundary, so that the shape of the surface determines the field as in the experiment.

The results of calculations of the post-implosion dynamics of the plasma in the Scyllac geometry are presented. Parameters for these calculations correspond to the actual full torus experiment. The plasma is initially at rest in a uniform torus centered in the discharge tube. The magnetic flux corresponds to the maximum field for the experiment. The plasma motion may be divided into two phases. During a fast dynamic phase, the plasma accommodates to the bumpy, helical stellarator fields, and thus experiences an inward force opposing the toroidal drift force. A quiescent phase is observed to follow. The current calculations are examining the effect of diffuse profiles on the dynamic phase.

A comparison between corresponding diffuse and sharp boundary calculations is made which concentrates on the $\ell = 0$ and $\ell = 1$ responses of the plasma column and the degree of toroidal force balance achieved. Several differences are noted. First, the diffuse plasma exhibits phase mixing effects so that oscillatory motions are rapidly damped. Second, toroidal forces are not balanced as well for a diffuse profile as they would be for a sharp boundary profile with corresponding fields.

Results of calculations are presented indicating plasma response to the $\ell = 0$ and $\ell = 1$ fields and the field parameters required to achieve approximate toroidal force balance for both diffuse and sharp boundary profiles.

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Poster Session 1A

MHD Stability Theory

"Kinetic" Theory of Internal Kink Modes*

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We have developed a theory of the internal m = 1 kink mode including the effects of finite gyro-radius, resistivity, and ion-ion collisions. When the last effect is neglected, analytical solution[1] of the problem can be given. The eigenvalue equation is:

$$\lambda = \lambda_H \left[\frac{1}{8} \lambda^{5/4} \left(\hat{\lambda} - i\hat{\lambda}_e \right)^{3/4} \left(\hat{\lambda} - i\hat{\lambda}_i \right)^{1/4} \left\{ \Gamma(x) / \Gamma(x + \tfrac{3}{2}) \right\} \right],$$

where, $x = \frac{1}{4}\left\{ \hat{\lambda}^{1/2} \left(\hat{\lambda} - i\hat{\lambda}_e \right)^{1/2} \left(\hat{\lambda} - i\hat{\lambda}_i \right)^{1/2} - 1 \right\}$, $\lambda = \gamma\tau_H$,

$\lambda_H = \gamma_{MHD}\, \tau_H$, $\lambda_{e,i} = -i\omega_{*e,i}\, \tau_H$, and $\hat{\lambda} = \lambda/\varepsilon^{1/3}$, etc. Here

$\varepsilon = \tau_H/\tau_R$, τ_H being a typical Alfven time, τ_R the resistive diffusion time, and ω_* the drift wave frequency.

The relevant growth rate can be considerably reduced compared to the one that is found when only resistivity is taken into account[2] and the mode acquires a frequency of oscillation that corresponds to a phase velocity in the direction of the electron diamagnetic velocity and its magnitude is consistent with that observed in the ATC and Pulsator devices. The effects due to transverse collisional viscosity are shown to become important above a realistic temperature threshold and appear to introduce a damping for the same mode.

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* Work supported in part by the U.S. Energy Research and Development Administration

1. B. Coppi, R. Galvão, R. Pellat, P. Rutherford and M. Rosenbluth - Paper presented at this meeting (1976).
2. B. Coppi in Propagation and Instabilities, W.I. Futterman (Stanford University Press, 1963) p. 70

1A-2

On Drift-Tearing and Driven Tearing Modes*

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Two problems connected with tearing modes in a resistive plasma have been investigated. In the first, we analyze drift-tearing instabilities without making the "constant-ψ" approximation near the mode rational surface. This permits coupling of the tearing mode to mixed (electrostatic-electromagnetic) short wavelength perturbations. The results indicate that the earlier "constant-ψ" approximation results are valid only in certain limits and that in other limits, coupling to the short wavelength modes might strongly modify the instabilities. In the second problem we study the time evolution of a stable $m = 3$ tearing mode in a tokamak, driven by the coupling of currents due to a saturating $m = 2$ island and the poloidal periodicity in the toroidal magnetic field. The slab model is used and a combination of analytical and numerical techniques is used to study the evolution of the $m = 3$ mode. The results show that highly asymmetric magnetic islands with very localized currents form about the $q = 3$ surface. These islands are smaller than the existing $m = 2$ islands by a factor of order $(a/R)^{1/2}$.

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1A-3

Instabilities of Straight Cylindrical Anisotropic Guiding Center Plasma

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Abstract

The effect of anisotropy on the equilibrium and the stability of the Guiding Center Plasma (GCP) in straight cylindrical geometry is examined. The anisotropy is introduced as a parameter $\alpha (\equiv P_{\parallel} / P_{\perp})$, which is a constant over the profiles. The GCP equilibrium equation is solved in terms of the parameter α, and the related equilibrium quantities are expressed as functions of α. For an assumed Tokamak-like magnetic field with shear, the numerical stability calculation is carried out to yield the envelope of the maximum growth rate vs. the longitudinal wave number k. The α-dependence of the growth rate of an eigenmode (m,k) is examined and the relation between the growth rate and the Suydam criteria is discussed.

Resistive Internal Kink Modes*

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For plasma parameters that are realistic for present day experiments we find that finite electrical sensitivity affects the description of the m = 1 internal kink mode in two important ways: a) the relevant growth rate is considerably increased and for parameters corresponding to marginal stability, according to the idealized MHD theory, instability is still found with a growth rate proportional[1] to $(\tau_H/\tau_R)^{1/3}\tau_H^{-1}$. Here τ_H is the appropriate poloidal Alfven time and τ_R the resistive diffusion time; b) the topology of the perturbed magnetic field differs from that resulting from the idealized MHD theory and involves the formation of a magnetic island.

A complete solution of the problem is provided both analytically and numerically. The relationship with tearing modes is analyzed and the relevance of the presented theory to the interpretation of disruptive instabilities is discussed.

[1] B. Coppi in Propogation and Instabilities in Plasmas, Ed. W. I. Futterman (Stanford University Press, 1973) p. 70

* Sponsored in part by the U.S. Energy Research and Development Administration

The Local Stability of Doublet.* D. R. Dobrott, General Atomic Company--The ideal and resistive interchange stability of free-boundary doublet equilibria is analyzed numerically and discussed in detail. The equilibria are obtained in a toroidal chamber of rectangular cross section (30 × 90 centimeters) to simulate DIIA. A limiter is used to form the free boundary. Flux dependent and independent current profiles are examined.

Stability against localized interchange is examined by the criteria of Mercier and Glasser, et al.; and results are interpreted in terms of the interplay among flux surface averaged quantities, such as magnetic well and shear, the remnants in the Mercier criterion of the "ballooning mode," and the spatial variation in the magnetic field energy density over the cross section. It is this variation which gives rise to the resistive modification of the Mercier criterion.

The code analyzes separately stability in the neighborhood of the magnetic axis and in the remainder of the plasma. In addition to details of the physics mentioned above, quantities such as maximum beta and current, critical safety factor and percent volume of chamber filled by the plasma are calculated. The code is used to compare optimized free-boundary equilibria in the 30 × 90 centimeter box for a variety of cross-sectional shapes, viz. circles, dees, and doublets.

*Work supported by the U. S. Energy Research and Development Administration, Contract E(04-3)-167, Project Agreement No. 38.

Finite Hybrid Elements to Compute the Entire Ideal 2D MHD Spectrum*

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The calculation of the MHD spectrum in a low-β configuration revealed itself to be a much harder problem than anticipated. In the 1D cylindrical case a so-called "spectrum pollution" was observed when linear finite elements were used for all three vector components in a Ritz-Galerkin expansion. The strong coupling between Alfven and fast modes made it impossible to obtain the unstable part of the spectrum with a reasonable number of mesh points. This difficulty was removed by a suitable choice of different basic functions for different vector components [1].

The same idea of choosing different basis functions for different components of the displacement vector was then tried in two dimensions. This approach, using regular finite elements, was pollution-free and the kink modes were well reproduced. But the weakly growing internal modes showed bad convergence behaviors. This problem was traced to a bad representation of the operator $\underline{B}\cdot\underline{\nabla}$.

A new numerical scheme, which we propose to call "finite hybrid elements", has been tried. For that purpose we extend the variational form by introducing new variables for the ψ and χ derivatives of the vector components and imposing auxiliary conditions. Instead of making a variation with three unknowns we vary seven unknowns which are linked through four auxiliary conditions. With an appropriate choice of finite basis functions, together with piecewise constant equilibrium and geometrical quantities, one is able to force all quadradic terms to be piecewise constant. This approach is quite similar to finite differencies applied to a variational form. In all straight elliptical low-β test calculations performed up to now with these finite hybrid elements, we reproduce the weakly growing modes with high accuracy already with a few mesh points. A general purpose toroidal code has been written and is in its final testing phase. Remarkable advantages of this simple code are efficiency in computing time and memory space.

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*Work supported by U.S. Energy Research and Development Administration Contract E(11-1)-3073, and the Swiss National Science Foundation.

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[1] K. Appert, D. Berger, R. Gruber, J. Comp. Phys. 18, (1975), 284.

Spectrum and Local Stability of MHD Equilibria with Flow

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The linear stability analysis of steady, cylindrically symmetric flows with no radial components, yields a real continuous spectrum with Alfvén and cusp parts similar to the static equilibrium case. An additional continuum arises from resonances between perturbation and equilibrium velocities. Suydam (local) stability exists if and only if one of the following conditions holds:

1. The shear of the equilibrium flow is "large enough" compared with the shear of the magnetic field.
2. A Suydam-like inequality.
3. A "peculiar" condition which depends on the connection coefficients (circuit relations) of a second order ODE with five singularities in the entire complex plane, and whose coefficients depend on the plasma profiles.

In the case of local instability, the unstable modes (ω) accumulate along two rays $\arg(\omega-\omega_0) = \theta$ or $\pi - \theta$ with θ depending on the global behaviour of the plasma profiles. The accumulation point ω_0 is the overlapping point of the three continuous spectra.

NONLINEAR IDEAL MHD EVOLUTION: VERIFICATION*

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We have used initial value codes to study the nonlinear evolution of unstable equilibria. The initial exponential growth saturates, but if the linear mode is localized, then the nonlinear convection is also localized.

A changing convection pattern (including convection of plasma current and B field) and motion of the velocity vortex cells are strictly nonlinear effects. The qualitative aspects of these results hold for a variety of geometries, mode numbers, q values and β's.

Extensive numerical tests have been performed including: convergence, stability, time reversal and greater precision. The results of these tests will be presented.

*Research sponsored by the U. S. Energy Research and Development Administration under contract with Union Carbide Corporation.

Two-Dimensional Time-Dependent Calculations of Axisymmetric Equilibrium and Stability of Tokamaks*

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A finite difference method[1] based on magnetic flux coordinates is applied to computing the dynamics of a plasma-vacuum system with tokamak parameters where phenomena on the MHD stability time scale is of interest. The method is used to compute the linear growth and the nonlinear saturation, if any, of axisymmetric instabilities[2] of noncircular cross section tokamak plasma-vacuum systems. Surface currents induced in the plasma play an important role in the nonlinear stabilization of some axisymmetric modes which are linearly unstable.

The addition of fictitous viscous force terms to our model provides a mechanism for dissipation which then enables us to compute equilibria which are stable with respect to axisymmetric modes.

*Work supported by U.S. Energy Research and Development Administration Contract E(11-1)-3073.

†On loan from Westinghouse Research Laboratories.

[1] S.C. Jardin, J.L. Johnson, J.M. Greene, R.C. Grimm, Bull. Am. Phys. Soc., 20, 1264(1975).

[2] M.D. Rosen, Phys. Fluids, 18, 482(1975).

Numerical Study of the MHD Spectrum in Tokamaks With a Noncircular Cross Section *

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The behavior of a class of exact tokamak equilibria is investigated in the context of ideal magnetohydrodynamics. The equilibrium solution allows for a general plasma cross section shape, restricted to systems with essentially flat toroidal current profiles. The spectrum of the eigenmodes is evaluated by extremizing the Lagrangian using a Galerkin procedure. The different branches of the spectrum are appropriately represented by expanding the eigenfunctions in terms of the global eigenfunctions of the corresponding straight limit. The perturbed vacuum field is expressed by a vector potential. The code is applied to various configurations with a fixed and with a free plasma boundary. The influence of ellipticity of the cross section and of the aspect ratio on the spectrum is discussed. Increasing the inverse aspect ratio in the equilibrium model treated does not provide a stabilization of free boundary modes, for values of the safety factor at the plasma surface less than six. For a fixed plasma boundary the stability limit agrees well with Mercier's criterion.

* Work supported by U. S. Energy Research and Development Administration contract E(11-1)-3073.

Influence of Finite Gyration Radius on the $m = 1$ Internal Mode in Tokamaks *

G. Laval, M. N. Rosenbluth and B. V. Waddell

ABSTRACT

We study the stability of the $m = 1$ hydromagnetic internal modes, taking into account resistivity, finite ion Larmor radius effects, as well as Hall and thermoelectric effects in Ohm's law. When the resistivity is large, we can neglect the small terms leading to the internal kink mode instability of ideal MHD. In this case, the F.L.R. corrections behave in an unusual way when compared with known results for $m \geq 2$ modes. Thus, for $m = 1$, the growth rate s is given by $s(s-i\omega^*_i)(s-i\omega^*_e) = \tau_R^{1}\tau_H^{-2}$, while for $m \geq 2$, we find $s(s-i\omega^*_i)(s-i\omega^*_e)^3 = \tau_R^{-3}\tau_H^{-2}$ where τ_R and τ_H are the usual characteristic resistive and MHD times. We recover also for $m = 1$ the characteristic $m \geq 2$ behavior if the internal kink mode is stable and if the resistivity is small enough. Finally, if resistivity is neglected, finite ion gyration effects stabilize the internal mode though an unstable residual growing mode persists whenever a small resistivity is added. Comparison with experimental results will be discussed.

*Research sponsored by the U.S. Energy Research and Development Administration, Grants No. E(11-1)3237 and E(11-1)3073.

The Princeton Equilibrium and Stability Package*

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Work is continuing on the computational program to determine the MHD spectrum and its associated eigenmodes for numerically determined tokamak configurations.[1,2] Comparison of the predictions of this model with those obtained by two other approaches[3,4] has been started with some qualitative agreement. This required the inclusion of an arbitrarily placed conducting wall in the code. At the same time, a formalism for incorporating the special flux constraints associated with the multivalued contribution[5] to the scalar potential in the vacuum region has been developed.

*Work supported by U.S. Energy Research and Development Administration Contract E(11-1)-3073 .

†On loan from Westinghouse Research Laboratories.

1R.C. Grimm, Bull. Am. Phys. Soc. II, 20, 1263(1975).

2R.C. Grimm, J.M. Greene, and J.L. Johnson, in "Methods in Computational Physics", (1976) Vol. 16, p. 253.

3W. Kerner, this session.

4R. Gruber, D. Berger, and F. Troyon, this session.

5R. Lüst and E. Martensen, Z. Naturforsch A15, 706(1960).

[Please place in same poster session as Kerner and Gruber *et al*.]

Nonlinear Growth of the Tearing Mode During the Skin Current Phase of the Tokamak Discharge*

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ABSTRACT

It has been shown both analytically and numerically that tearing modes have nonlinear growth times only a few hundred times smaller than the resistive time. This result is for the case where the poloidal mode number is 2 or greater and where the current profile is peaked in the center of the tokamak column. It has been suggested that this behavior does not occur for a skin current profile typical of the early stages of a tokamak discharge. Instead, it has been proposed that the fast growth of the islands in the linear phase continues into the nonlinear stage and that ultimately the current profile flattens and possibly penetrates the center of the column. A numerical calcualtion has been performed to decide this question, and the results of this calculation will be presented.

*Research sponsored by the U.S. Energy Research and Development Administration, Grants No. E(11-1)3237 and E(11-1)3073.

Effect of Hot Electrons on Localized MHD Instabilities

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Although a necessary condition for local macroscopic stability is violated in the Elmo Bumpy Torus plasma, no resultant instabilities are observed. Previous analysis using the Vlasov equation and an inhomogeneous slab geometry indicated stabilizing effects on hot electron drift modes ($\omega \sim \omega_{de}$).[1,2] Here localized MHD modes ($\omega \sim -\sqrt{g\rho'/\rho}$) in an inhomogeneous slab with gravity are discussed, using the Vlasov equation and considering a three component plasma consisting of warm ions, hot electrons, and cold electrons. The ratio of hot electron density to ion density is denoted by δ. Analysis of this geometry using the MHD equations shows that if $g\rho' < 0$, the most unstable localized modes are electromagnetic and have $k \cdot B = 0$.[3] For the Vlasov analysis we assume $k \cdot B = 0$, and with $\delta = 0$ recover substantially the MHD growth rates. As δ increases the equilibrium is modified, but if $\delta < ka_i$ (a_i is the ion Larmor radius) and $\delta < \beta_{ion}/\beta_{hot\ electron}$, the form of the dispersion relation is unchanged. The growth rates are modified, since the coefficients depend on the equilibrium, and the plasma can be stabilized to local modes. This may be called the rigid hot electron regime. As δ further increases ω^2 becomes complex, Re ω approaches ω_{de} (the case previously considered), and the MHD equations are no longer appropriate for studying localized modes.

1. G.E. Guest, C.L. Hedrick, D.B. Nelson, Phys. Fluids 18, 871(1975)
2. H.L. Berk, private communication
3. W.A. Newcomb, Phys. Fluids 4, 391(1961)

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Axisymmetric MHD Instabilities of Doublet-type Plasma Configurations.

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The stability of Doublet-type plasma configurations with respect to axisymmetric perturbations has been studied using a surface current model. Instability was found with respect to slip modes /1/ which are stable in comparable plasmas with standart cross-sections (elliptical etc.). In Doublet-type configurations a sub-class of these instabilities exhibits a striking difference between the middle section of the plasma and the lateral wings, the perturbations suffering a discontinuous (or, rather, a rapid) transition across the cylindrical interfaces. These perturbations tend either to tear off the wings from the middle or to push them into it. Simultaneously, the middle section moves in a vertical and / or radial direction. It would appear that this discontinuous behaviour is tempered by wall stabilization.

/1/ E. Rebhan, A. Salat, VIIth Europ. Conf. on Contr. Fusion and Plasmaphysics, Lausanne 1975, p. 101

Stability of the Elmo Bumpy Torus

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The stability behavior of EBT has not been explained satisfactorily. Within the macroscopic theory it was found that a local stability condition is grossly violated[(1)], but this result was disregarded[(2)] with the argument that all the resultant modes are highly localized and therefore are stabilized by non-ideal effects[(3)]. However, this argumant need not apply to all modes because it was realized more recently[(4)] that violation of local criteria[(5)] also signals the presence of gross instabilities. The latter have not been studied even though they should, at least qualitatively, be correctly described by MHD. In the present paper we calculate the growth rates of gross MHD modes in simple EBT-like equilibria. Various assumptions (viz., neglect of toroidal curvature, expansion in geometrical parameters, scalar pressure, step profiles in pressure and density) make this problem amenable to a fully analytical treatment. The results will be compared with experimental findings, and applied to future operating regimes.

(1) D.B. Nelson, G.O. Spies, C.L. Hedrick (to be published)
(2) G.E. Guest, C.L. Hedrick, D.B. Nelson, Phys. Fluids 18, 871 (1975)
(3) D.B. Nelson, this meeting
(4) G.O. Spies, Phys. Fluids (March 1976)
(5) G.O. Spies, Phys. Fluids 17, 400(1974)

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THE SHARP-BOUNDARY SCREW PINCH: KINETIC CORRECTIONS TO MAGNETOHYDRODYNAMICS ACCORDING TO VLASOV-FLUID THEORY*

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We have studied properties of Freidberg's[1] Vlasov-fluid model for a sharp-boundary screw pinch in which the ion Larmor radius is small but non-zero and in which the equilibrium ion velocity distribution is isotropic.

A differential equation governing $\vec{\xi}_1(\vec{r})$ is derived and solved. We derive formulas exhibiting the $\vec{\xi}_1$-dependence of perturbations in the ion distribution function, in the ion number density, in the ion fluid velocity, in the pressure tensor, and in the heat flux vector. A dispersion relation is obtained. Comparisons are made with ideal magnetohydrodynamics (MHD).

Long-wavelength MHD instabilities with $m \geq 2$ are finite-Larmor-radius (FLR) stabilized in the Vlasov-fluid model.[2] For $m = 2$, we find that the instability is extinguished at an ion temperature equal to one-fourth of that given by Freidberg's analysis. This result is in reasonable agreement with recent numerical results of Lewis and Turner[3] as well as with recent experiments at Los Alamos and Garching. The result may facilitate simultaneous FLR stabilization of $m = 2$ and wall stabilization of $m = 1$ instabilities in near theta-pinch devices, such as Scyllac.

For short-wavelength instabilities, we have analyzed the case $m > 0$. We find that the domain of marginal stability of the Vlasov-fluid model is the same as that of ideal MHD, in agreement with the result of Freidberg.[1] The ratio of the Vlasov-fluid growth rate to the ideal MHD growth rate decreases to zero as marginal stability is approached. We also have solved numerically the Vlasov-fluid dispersion relation for the eigenfrequencies of short-wavelength perturbations.

1. J. P. Freidberg, Phys. Fluids 15, 1102 (1972).
2. Leaf Turner, "Finite-Larmor-Radius Stabilization in a Sharp-Boundary Vlasov-Fluid Screw-Pinch," Los Alamos Scientific Laboratory Report #LA-UR-76-397, submitted for publication.
3. H. Ralph Lewis and Leaf Turner, "Finite-Gyroradius Stabilization of Instabilities in Sharp-Boundary Screw Pinches," submitted to this conference.

*Work performed under the auspices of the U.S. ERDA.

Nonlinear Growth of the m = 1 Tearing Mode*

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ABSTRACT

Numerical results are presented for the nonlinear evolution of the tearing mode with poloidal mode number one. Nonlinearly, the mode continues to grow exponentially at approximately the linear growth rate until it flattens the toroidal current inside the singular surface and increases the safety factor to unity at the plasma center. The hypothesis that the mode causes the internal disruption in tokamaks is supported by the fact that the time scale for the process agrees with experiment.

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SIMULATION OF RESISTIVE TEARING MODES IN TOKAMAKS

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The effects of resistive tearing modes upon Tokamak startup, oscillations, and current penetration have been investigated. Utilizing a 1-D diffusion time scale MHD code we follow the evolution of the singular surfaces $\vec{k} \cdot \vec{B} = 0$. Then, using the method of Furth, et. al., (1) we solve for the wave functions and growth rates of the unstable modes. A quasi-linear analysis is used in conjunction with estimates of the nonlinear limits of the instability to compute diffusion coefficients and the effects upon $\langle J_z \rangle$ and $\langle B_\theta \rangle$ due to the resistive modes. The relevance of these calculations to PLT, ST, and TFR will be presented.

[1] H. P. Furth, P. H. Rutherford, and H. Selberg, Phys. Fluids, 16, 1054 (1973).

*Work supported by U. S. Energy Research and Development Administration.

Neoclassical Theory of The Mini-Disruptions

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The new neoclassical theory for impure Tokamak plasmas[1], which allows for the electrostatic potential variation on a magnetic surface, $\Phi(\theta)$, and retains electron viscous forces, leads to a quintic in the radial electrical field (E_r) for the equilibrium condition $\partial V_\theta/\partial t = 0$. This equation has been solved for the central core of the discharge after the m = 1 instability has occurred, making the radial gradients of n, T_e and T_i small, so that the only forces tending to make $V_\theta \neq 0$ are the electron viscous forces caused by the current driven by E_φ. For nearly all values of $\sigma_{\parallel} E_\varphi$ there is only one real root for E_r, but in a narrow range of $\sigma_{\parallel} E_\varphi$ just below $\sigma_{\parallel} E_\varphi = 2n_e e v_{Ti}$ (the "critical range"), there are three real roots. Below this critical range the solution for E_r has only moderate magnitude and $e\Phi(\theta)/T < r/R$. The neoclassical equations predict that $\partial T_e/\partial t$ should be positive and the conditions correspond to the slow rise of the observed sawtooth. When T_e reaches the value corresponding to the upper side of the critical range, the incremental resistivity $(\partial j_\varphi/\partial E_\varphi)$ becomes abruptly negative and an instability causes a rapid change to a new equilibrium with $e\Phi(\theta)$ increased by a factor of about 4. In this new equilibrium $\partial T_e/\partial t$ is negative because of a much larger neoclassical heat transfer from electrons to ions and because the m = 1 region expands. When T_e reaches the lower limit of the critical range a second instability changes the equilibrium back to the original conditions.

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[1] R. D. Hazeltine, and A. A. Ware, Phys. Fluids, to be published.

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The m = 1 Resistive Kink Mode

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Abstract

A variational solution of the equations which describe MHD resistive instabilities is discussed. This obviates the necessity to invoke the "constant Φ approximation". A dispersion relation is obtained using the frequency dependent Ohms law derived by Hazeltine et.al,. The results of Hazeltine et.al,[1] on the kink-tearing modes are extended to include m = 1 kink perturbations.

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1. R.D. Hazeltine, D. Dobrott, and T.S. Wang, Phys. of Fluids 18, 1778 (1975).

*This work supported by U.S. ERDA contract E-(40-1)-4478.

Poster Session 2A

Auxiliary Plasma Heating, Waves Simulation

Thermal Transport in RF (Lower Hybrid) Heated Plasmas.* T. M. Antonsen Jr. and E. Ott, Laboratory of Plasma Studies, Cornell University.--We determine the temporal evolution of the temperature profile of an rf heated plasma torus. Our model includes the following effects: (1) inward energy transport by externally launched lower hybrid waves which Landau damp and heat the plasma as they propagate from the plasma edge to the vicinity of the lower hybrid resonance where the remainder of their energy is deposited; (2) quasilinear plateau formation and the competing effect of Fokker-Planck collisions; (3) electron and ion thermal conductivity; and (4) temperature equilibration. The implications of various models of the thermal conductivity (neoclassical, psuedoclassical, and anomalous) and of the heat deposition near the resonance (mode conversion, parametric instabilities, etc.) are investigated. It is found that quasilinear flattening is effective in reducing the Landau damping of the lower hybrid waves and thereby prevents large amounts of energy from being deposited in the outer regions of the plasma.

*Research supported by ERDA.

Electron Heating in Tokamaks with RF Power at the Lower Hybrid Frequency*

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ABSTRACT

The penetration of RF fields into a high temperature tokamak plasma may be strongly inhibited by electron Landau damping[1]. We present a theoretical analysis and computations which show that this can in fact be used for supplementary heating of the electrons and/or for controlling the current profile in a tokamak plasma.

A two-dimensional analysis of the power dissipated by electron Landau damping of the fields emanating from a finite length waveguide array structure at the wall will be given. This shows that with proper care in the design of the array as much as 90% of the incident power can be absorbed in the plasma. Appreciable heating of the whole plasma can be obtained by using several such waveguide arrays at the plasma wall so that the heating fields effectively fill the plasma cross section.

A quasi-linear analysis will be presented to show how the RF energy absorption affects the electron velocity distribution function. With a large number of arrays the heating mechanism can be regarded as due to stochastic acceleration of the electrons as they traverse the electric field regions created by the various arrays.

A nonlinear analysis that considers the effects of trapping and possible enhanced diffusion will also be described.

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*Work supported by ERDA-DCTR Contract Number E(11-1)-3070.

[1]A. Bers in Proc. U. S. - Australian Workshop on Plasma Waves, R. C. Cross ed., University of Sydney, Sydney, Australia, Feb. 1975, pp. 5.1-6.

The Effect of Low-Frequency Random Density Fluctuations on the Raman-Backscattering Process Near the Upper Hybrid Frequency.* V. S. Chan, General Atomic Company--The application of RF near the hybrid frequencies for plasma heating in a fusion device has received great interest recently. In utilizing the upper hybrid frequency, it has been shown that in a noise-free plasma, Raman-backscattering process can occur which can result in the radiative transport of electromagnetic pump energy away from the plasma. In real-life machines, there are always present low-frequency random density fluctuations. These fluctuations may stabilize the above-mentioned process by raising its threshold hence reducing the radiative loss. Such an effect is investigated and illustrative numerical results are presented.

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New Three-Dimensional Simulation Models for Cylindrical and Toroidal Plasmas*

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A new class of three-dimensional particle simulation models has been developed which is suitable for simulating large cylindrical and toroidal devices such as tokamaks or pinch devices. The model makes use of the eigenfunction expansion in the homogeneous toroidal or axial direction while the multipole expansion around the nearest grid point is employed using a two-dimensional spatial grid introduced on the poloidal cross section for solving the Maxwell equations.

The model has several attractive features for simulating large devices from physical point of view as well as for numerical computation. By eliminating the use of spatial grid while keeping the reasonable number of long wavelength modes in the axial direction, required grid size for computation is reduced easily by more than an order of magnitude compared with the conventional grid approach in three directions, since the experimental device is much larger in the axial direction than the transverse dimension. The use of much smaller number of modes compared with the number of grid points in the axial direction is justified since the most important physics are often associated with the modes of $k_\perp \gg k_\parallel$ and therefore the variation along the field line is very slow. This means the scale length in the axial direction is much longer than that for the transverse direction and this naturally suggests the grid-mode hybrid approach. One would waste time and memory by keeping three dimensional grid. The effects of short wavelength modes are built in through the Monte-Carlo collision operator for the Fokker-Planck collisions.

Both cylindrical and toroidal electrostatic codes have been developed and tested for a thermal plasma. Ions are pushed using the full Lorentz force while electrons are assumed to be guiding center particles. Test of the results is quite satisfactory, in particular, the thermal spectrum for the fluctuations are in good agreement with theory. Self-consistent magnetic field necessary for toroidal pinch simulation is being built in and the results will be reported.

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COMPUTER SIMULATION OF RADIO FREQUENCY CONFINEMENT OF A PLASMA NEAR CRITICAL DENSITY*

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The time evolution of an initially uniform slightly underdense ($\omega_e \lesssim \omega$) plasma in a resonant cavity has been studied using a two-dimensional fully electromagnetic simulation code. The following physical picture appears to describe the behavior. The pondermotive force due to the nonuniform cavity fields squeezes plasma into electric field nodes. The density growth in time at the nodes is proportional to t^2. Such behavior is explained by the nonlinear theory of stimulated Brillouin scattering.[1] The density growth saturates in a way which depends on the intensity of the applied cavity fields. For low field levels, saturation occurs due to a balance between the energy density of plasma and cavity fields. For higher field levels, saturation by phase space breaking occurs. In addition, for the higher field levels, non-thermal high energy tails form in the electron and ion distribution functions which behave approximately as $\mathrm{EXP}(-a|v|)$ where a is a constant, in agreement with theoretical results of Bezzerides and DuBois.[2] The peak asymptotic particle densities are found to depend on the initial density n_o and the fractional volume of the cavity occupied by the confinement regions. In this case n_{max} $(t \to \infty) \simeq 2n_o$. In light of these results, we will discuss the viability of RF confinement to yield plasmas of thermonuclear interest.

*Work performed under the auspices of the United States Energy Research and Development Administration and the National Science Foundation.

1. D. W. Forslund, J. M. Kindel, and E. L. Lindman, Phys. Fluids 18, 1002 (1975).

2. B. Bezzerides and D. F. DuBois, Phys. Rev. Lett. 34, 1381 (1975).

PHYSICS OF SLOW ALFVÉN WAVE HEATING

by

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ABSTRACT

An externally applied oscillating magnetic field (at a frequency near one MHz for typical tokamak parameters) resonantly mode converts to the kinetic Alfvén wave, the Alfvén wave with the perpendicular wavelength comparable to the ion gyroradius. The kinetic Alfvén wave, while it propagates into the higher density side of the plasma after the mode conversion, dissipates due to both linear and nonlinear processes and heats the plasma.

In a collisional regime both electrons and ions are heated almost at an equate rate, while in a collisionless regime, but yet if $\beta \lesssim 0.1$, electrons are heated linearly by the Landau damping while ions are heated nonlinearly by decay to the ion acoustic wave. When β exceeds 0.1, ion Landau damping becomes comparable to the electron Landau damping and both species can be heated linearly again at approximately the same rate.

The linear heating rate in this scheme is faster than that in the transit time magnetic pumping (magnetosonic wave heating) by a factor of β^{-1}.

Nonlinear heating occurs when the applied field exceeds a threshold of typically several tens of gauss. If $T_e \gtrsim 5T_i$, resonant decay to the ion acoustic wave, while if $T_e \lesssim 5T_i$, nonlinear ion Landau damping heat ions respectively. In any case total absorption occurs ($Q \approx 1$) thus if a magnetic field of 50 gauss effective amplitude is applied approximately ten mega joule per cubic meters of energy can be deposited in one second into the plasma.

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The Angular Dependence of Ion Cyclotron Instability In a D-T Beam-Plasma System*

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Abstract

A code has been developed to study the stability of a variety of waves in a magnetized ion beam plasma system. The beam injection velocity, v_b, is assumed to be $v_{thi} < v_b < v_{the}$, consistent with TFTR parameters. In this approximation the Fokker-Planck equation for the beam distribution function may be expanded in Legendre Polynomials. A non-standard representation (in terms of Chebyshev polynomials) for the delta function source was chosen for optimal convergence. The distribution function is then used to compute the energy transfer between the beam particles and the wave under study, in terms of the functions, $\epsilon_b^{\ell}(k_\perp v_\parallel)$.[1] This is compared to the energy loss due to backguard plasma ion and electron Landau damping to obtain a beam density threshold and growth rate.

The code is applied to the obliquely propagating ion cyclotron wave with the $\omega < k_\parallel v_{the}$. The stability analysis is performed for a variety of injection angles, beam energies, and other source function parameters. The three lowest order resonances for an ion cyclotron wave are: a wave doppler shifted up or down to the beam cyclotron frequency ($\ell = \pm 1$) and a Landau resonance ($\omega = k_\parallel v_b$). Two mechanisms appear to control the angular dependence of the threshold. First, as the injection varies from parallel to perpendicular the typical size of the velocity gradients in the beam distribution function diminishes, which tends to increase the threshold for instability. Secondly, varying the angle from parallel to perpendicular moves the region of maximum velocity gradients to lower resonant velocities, tending to decrease the threshold. The competition between these two effects is evaluated for the three resonances. Thresholds and maximum growth rates are presented.

1. L.P. Mai and W. Horton, Jr., Physics of Fluids 18, 356, 1975.
*This work supported by U.S. ERDA Contract E-(40-1)-4478.

Nonlinear Propagation of Lower Hybrid Waves in Two Dimensions, and Three-Dimensional OTS and Filamentation Instabilities*

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ABSTRACT

The description of two-dimensional steady-state propagation of lower hybrid waves in a homogeneous plasma by the modified Korteweg-deVries (mKdV) equation[1] requires the assumption that the amplitude of the potential at frequency ω be real. The condition that power is flowing into the plasma from a plane boundary requires that the amplitude be complex and thus invalidates the applicability of the mKdV equation to the problem of propagation of lower hybrid wave energy from an external source.

If the assumption of reality is removed, the resulting equation, in which the nonlinear term is $(\phi^2\phi^*)_y$, has several properties that are different from those of the mKdV equation. Specifically, it is thought that this equation does not have soliton-like solutions.

To gain further understanding of the nonlinear propagation of lower hybrid wave energy into a plasma, including the $\underline{E}\times\underline{B}$ three-dimensional effects generally considered dominant, the problem of the oscillating two-stream and filamentation instabilities excited by a finite wavelength lower hybrid pump wave is investigated. Some results of this investigation are presented.

*Work supported by ERDA-DCTR Contract Number E(11-1)-3070.

[1] G. J. Morales and Y. C. Lee, Phys. Rev. Letters 35, 930 (1975).

PARAMETRIC EXCITATION OF ALFVEN AND OBLIQUELY PROPAGATING ACOUSTIC WAVES*

by J. N. Kappraff, New Jersey Institute of Technology, J. Tataronis, Courant Institute of Mathematical Sciences

Nguyen The Hung[1] has studied the nonlinear interaction of a large amplitude Alfven wave acting as a pump to excite another Alfven wave and a sound wave in the limit of low-beta using coupled mode theory. He assumed that the sound wave propagates parallel to a homogeneous magnetic field and sound a dispersion relation for the waves under consideration.

We have extended Hung's work to interactions of two Alfven waves with an ion-acoustic wave that propagates obliquely to a homogeneous magnetic field as a first step towards the accurate calculation of the threshold conditions for parametric instabilities of Alfven waves in inhomogeneous plasmas. We transformed the MHD equations to normal mode by first writing the equations as a 7 x 7 symmetric matrix and then diagonalizing this matrix by making a unitary change of coordinates. The normal mode equations were coupled by including the quadratic terms in the limits of both low beta and incompressible plasmas. We then applied coupled mode theory to obtain the dispersion relations that govern the growth of the parametric instabilities.

Comparing our results to Hung's, we find that the dispersion relations for low beta differ only in a multiplication factor of $\cos^3\theta$ that appears in our definition of the coupling coefficient, K. As a result, we find that our results agree for ion-acoustic waves propagating parallel to the magnetic field whereas for ion-acoustic waves propagating at right angles to the magnetic field, both threshold intensity for excitation of instabilities, and maximum growth rates vanish.

[1] Nguyen The Hung,, J. Plasma Physics (1974) Vol. 12, part 3, pp. 445-453

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Lower Hybrid Ray Penetration into a Tokamak: Electromagnetic and 3-D Effects*

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ABSTRACT

It is commonly thought[1] that the penetration of RF fields into a tokamak plasma from a localized source at the wall proceeds along a resonance cone ray which is independent of $n_{\parallel} = ck_{\parallel}/\omega$ as long as $n_{\parallel}$ is greater than the accessibility $n_{\parallel a} = (1 - \omega^2/\Omega_e\Omega_i)^{-1/2}$. This is strictly true only in the electrostatic approximation for a cold plasma. We show that for typical tokamak plasma parameters this approximate description of the ray is only valid for $n_{\parallel} > 3$. For $n_{\parallel a} < n_{\parallel} < 3$, the electrostatic and electromagnetic branches are sufficiently coupled to modify the ray trajectory considerably and make it a function of $n_{\parallel}$. The consequence of this is that RF energy in this spectral range does not penetrate the plasma so readily, and is spread toward the wall over an extent which is much larger than would be expected from the electrostatic ray structure. Recent experiments on Alcator and ATC apply waveguide field structures that have an important energy content in this regime of $n_{\parallel}$. The resultant spread of the RF fields is shown to eliminate strong, nonlinear self-focusing effects, and to facilitate parametric decay processes which now can occur closer toward the wall.

We present a three-dimensional WKB analysis[2] of the ray penetration into a tokamak plasma and magnetic field geometry. The effect of the rotational transform is to wrap the ray around the minor cross section as it penetrates the plasma density profile. This filling of the cross section with RF fields, which is important in getting effective heating, is a strong function of applied frequency. Results of computations will be presented.

*Work supported by ERDA-DCTR Contract Number E(11-1)-3070.

[1] R. J. Briggs and R. R. Parker, Phys. Rev. Letters 29, 852 (1972).

[2] A. Bers in Plasma Physics - Les Houches 1972, Gordon and Breach Science Publishers (1975) p. 208.

Monte Carlo Calculations of Neutral Beam Heating of Tokamaks*

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The effects of neutral beam heating of tokamaks are modeled using a direct simulation of the neutral beam as a collection of fast neutral atoms and fast ions. The neutral atoms are injected into the torus by following individual particle paths and generating fast ions by a Monte Carlo method. This allows the inclusion of such effects as divergence of the beam and the injector geometry. The time behavior of these fast ions is determined by integrating the guiding center orbit equations of each particle in the distribution. The guiding center orbit equations include the toroidal curvature and gradient B drifts, and field ripple. The particles are pitch angle scattered by a Monte-Carlo method, and are slowed down classically. In this way, losses of particles due to orbit effects from large banana excursions and trapping in the field ripple can be estimated for such experiments as TFR, future experiments such as TFTR, and the proposed ripple injected tokamak.[1]

*Work supported by U.S. Energy Research and Development Administration Contract E(11-1)-3073.

[1] D.L. Jassby, R.J. Goldston, Enhanced Penetration of Neutral-Beam Injected Ions by Vertically Asymmetric Toroidal-Field Ripple, MATT-1206, Princeton University, 1976.

Anomalous Pitch Angle Scattering of Tangentially Injection Fast Ion Beams*

L. P. Mai and Wendell Horton, Jr.

From the previous study of electrostatic and electromagnetic instabilities of the magnetized fast ion distributions produced by tangential neutral beam injection we have found that the lowest beam density threshold for instabilities occurs for the ion cyclotron wave driven by the $T_{b\perp} \ll E_b$ anisotropy. The stability criterion is sensitive to the velocity spread of the beam particles with energies just below the injection energy. The resonant transfer of energy from parallel kinetic energy to the wave occurs through the $\ell = -1$ cyclotron resonance. The fastest growing mode has $k_{\parallel} \simeq (\omega_{ci} + \omega_{cb})/v_b$ and $k_{\perp} \cong \omega_{cb}/v_{b\perp}$ where $E_{b\perp} = \frac{1}{2} m_b v_{b\perp}^2$ is the typical perpendicular kinetic energy of a beam ion in the resonant regime. The threshold beam density and the corresponding critical beam to thermal plasma energy density ratio Γ_c of the mode are calculated from the transient solution of the Fokker-Planck equation. For the strong current, pulsed injection system, the threshold density $(n_b/n_e)_c$ saturates at about 10% when E_b/T_e is higher than 40-50, since the background ion damping is then negligible compared to the electron damping which is nearly constant. The quasilinear reaction of the resonant particles is dominated by diffusion in pitch angle so that particles increase their perpendicular kinetic energy. In cases where instability occurs the anomalous scattering from the fluctuations will dominate over the classical scattering and stabilize the system in a small fraction of slowing down time.

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Guiding-Center Calculation of Neutral Beam Heating for Tokamak

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A three-dimensional toroidal code for studying neutral beam heating of an axisymmetric tokamak has been developed. The code follows the guiding-center motion of thousands of beam ions in a torus with concentric magnetic surfaces. The background plasma is assumed to be Maxwellian described by macroscopic density and temperature functions. Slowing-down and pitch-angle scattering of the beam ions on the background plasma are incorporated by a Monte-Carlo method using the local Fokker-Planck coefficients. Several features of practical interest include beam penetration, velocity-space loss region, inhomogeneous plasma density and temperature, etc. The code is capable of simulating neutral beam heating such as planned for PLT and TFTR using a reasonable amount of computing time. The computed results for these cases will be reported. In addition we have studied by simulation the ion cyclotron instability and anomalous pitch-angle scattering, making use of the beam ion distribution calculated from the above code. The information on the anomalous pitch-angle scattering is fed back into the guiding-center code to simulate anomalous beam energy loss.

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Influence of the Two-Ion Hybrid Resonance On Minority Species Heating in a Tokamak*

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We consider the effect of the two-ion hybrid resonance on fast magnetosonic wave minority species heating in a tokamak. The collisionless hot plasma dispersion relation is solved for fast magnetosonic waves for $k_\perp >> k_{||}$. Due to the variation of the tokamak toroidal magnetic field with major radius, the two-ion hybrid resonance lies between the fundamental cyclotron resonance for the tritium (T) and deuterium (D) ions. A solution of the hot plasma dispersion relation for $\lambda = k_\perp^2 \rho_D^2/2$ as a function of the ratio of wave generator frequency to local deuteron gyrofrequency (ω/ω_{CD}) is obtained. For low $k_{||}$, the two-ion hybrid resonance and deuteron cyclotron resonance are separated by a region of wave evanescence. There is also a region of fast wave confluence with the electrostatic ion Bernstein mode near the two-ion hybrid resonance. We consider concentrations of the minority deuterons $n_D/n_e \leq 20\%$ for the TFTR design ($n_e = 10^{14}/cm^3$, $T_D = 4$ keV, $B_0 = 50$ kG, a=1 m, and R=2.5 m). In this regime the two-ion hybrid resonance lies close to that of the deuterium minority ion cyclotron resonance and both absorption processes must be considered. A modified Budden model is used to show that considerable tunneling through the evanescent zone and absorption of the fast wave energy by the two-ion hybrid resonance can be expected. For larger values of $k_{||} = 6\ m^{-1}$ determined by the wave launching structure, the evanescent region can be eliminated and mode conversion can be made negligible due to the very narrow region of wave confluence. It is shown that the inclusion of the two-ion hybrid resonance absorption can dominate that due to fundamental ion cyclotron absorption and considerably decrease the plasma cavity Q for the fast wave toroidal eigenmodes. The hybrid resonance can be expected to have a considerable impact on the TFTR fusion reaction rate. Its importance in considering the effect of a significant impurity ion concentration on wave absorption is also evident.

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FOKKER-PLANCK STUDIES OF THE C I T*

A. A. Mirin, M. E. Rensink, D. L. Jassby

A time dependent two-dimensional multispecies Fokker-Planck code is used to study the Counterstreaming Ion Torus (C.I.T.). The time derivative $\partial f_a/\partial t$ of the distribution function f_a is written as the sum of a Fokker-Planck collision term, a source term, a charge exchange loss term and a term expressing finite particle and energy confinement times.

The loss rate for the ions is given by

$$\frac{\partial f_a}{\partial t}\Big|_{loss} = \frac{-f_a(v,\theta,t)}{\tau_p(v)} ,$$

where $\tau_p(v) = \tau_{min} + (mv^2/3T_e)^{10}$ and T_e is the electron temperature. Ions with energies less than 3/2 T_e will have a confinement time of approximately τ_{min}, while higher energy ions will not be lost at all.

Parameter studies are carried out in which we vary the injection currents, injection energies, ion particle confinement times, electron energy confinement time, angular source width, background gas concentration, and the impurity type and concentration.

For co-injected 60 KeV D^+ beams and counter-injected 90 KeV T^+ beams, we obtain, in the absence of neutrals and impurities, a value of Q = 6 provided we specify a minimum ion confinement time (τ_{min}) of 300 msec. and an electron energy confinement time (τ_e) of 300 msec.

We make the following general conclusions:

a) For $10 \leq \tau \equiv \tau_{min} \equiv \tau_e \leq 300$ (msec), Q is a strongly increasing function of τ.

b) Background neutral gas causes a loss of particles due to charge exchange. The mean ion energy rises, but there is a net decrease of Q.

c) Q is drastically reduced when impurities are present in the system. The type of impurity and the Z-effective are both significant.

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The Nonlinear Coupling of Lower-Hybrid Radiation at the Plasma Edge

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In the lower-hybrid heating scheme there are three spatially separated regions within which widely different physical processes take place. Firstly, one encounters the edge of the plasma, where an evanescent electromagnetic wave mode-converts into a cold plasma wave. Then there is a large region of increasing density through which the mode-converted wave must propagate. Finally, there is the region near the center of the plasma where this wave meets the lower-hybrid resonance. Numerous linear studies of processes in all three regions have been made, and a few nonlinear investigations have appeared recently in the literature. An item of importance to all these investigations is the coupling efficiency of the plasma in the region where the frequency of the radiation matches the local electron plasma frequency. The present study is concerned with this question and in particular with the effects produced by nonlinear changes in the density profile on the mode-conversion process. Such nonlinear density changes can be triggered readily by the large energy density associated with the RF radiation applied at the plasma edge. The self-consistent slow time evolution of the nonlinear mode-conversion problem is formulated in terms of a set of coupled partial differential equations whose numerical study indicates the existence of a significant reduction in the amplitude of the mode-converted waves.

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A QUASI-NEUTRAL HYBRID MODEL FOR MULTI-DIMENSIONAL PLASMA SIMULATION*

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A hybrid model (kinetic ions, fluid electrons) is often the best compromise between physical realism and computational practicality. Various assumptions, such as massless electrons or small electron gyroradii, lead to several distinct hybrid models. For high-β plasmas, the quasi-neutral or small Debye-length approximation is usually appropriate, and one-dimensional simulation codes employing this approximation while retaining other electron inertial effects have been reported (1,2,3). We have formulated the quasi-neutral approximation in a form applicable to initial-value problems in two- or three-dimensions, and have implemented the model in a two-dimensional non-radiative hybrid simulation code. The essential observation in making the multi-dimensional generalization is that $n_e \approx n_i$ implies $\nabla \cdot J_e \approx \nabla \cdot J_i$. Using the subscripts ℓ and t to denote respectively the curl-free and divergence-free parts of a vector field, this requirement and the generalized Ohm's law for each species gives the equation

$$\left[\left(\frac{q_e^2}{m_e} - \frac{q_i^2}{m_i}\right) nE\right]_\ell = -\left[2\nabla\cdot\left(\frac{q_e}{m_e} K_e - \frac{q_i}{m_i} K_i\right) + \left(\frac{q_e}{m_e c} J_e - \frac{q_i}{m_i c} J_i\right) \times B\right]_\ell .$$

This equation replaces Poisson's equation in the quasi-neutral limit. It is combined with the appropriate form of Faraday's law to completely determine the electric field. The present code uses the non-radiative or Darwin limit of that law (4). An iterative technique has been found which provides a solution to these equations. When used in a diagnostic sense, this method reproduces with high precision the electric field obtained in a high-β particle simulation (5) in which Poisson's equation is solved explicitly. In the hybrid code, the ions are advanced in the usual way, and the moments of its distribution function accumulated. The electron density is equated to the ion density, and the longitudinal part of the electron current is advanced in accord with that of the ions. The transverse part of the electron current is advanced by using the transverse part of the generalized Ohm's law

$$\left(\frac{\partial J_e}{\partial t}\right)_t = \left[\frac{2q_e}{m_e} \nabla\cdot K_e + \frac{q_e}{m_e} nE + \frac{q_e}{m_e c} J_e \times B\right]_t .$$

This code has been applied to idealized pinch implosions in slab geometry. The code is numerically stable; however, improved differencing techniques for this equation are under study.

(1) D. W. Forslund and J. P. Freidberg, Phys. Rev. Lett. 27, 1189 (1971).
(2) R. J. Mason, Phys. Fluids 13, 1082 (1972).
(3) P. C. Liewer, Proc. Seventh Conf. on Num. Sim. of Plas., 202 (1975).
(4) C. W. Nielson, H. R. Lewis, and T. A. Oliphant, Proc. Meet. on Th. Aspects of CTR, 63 (1974).
(5) D. Winske and D. W. Hewett, Phys. Rev. Lett. 35, 937 (1975).

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GENERATION AND COLLAPSE OF TWO DIMENSIONAL LANGMUIR SOLITONS*

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J. Denavit, Northwestern University.

We have studied numerically the time evolution of two dimensional Langmuir solitons using Zakharov's model equations(1) with the inclusion of approximate electron Landau damping and the addition of an external pump field.(2)

In the first sequence of studies we initialize a planar soliton which is unstable to transverse perturbations longer than the soliton width. Without Landau damping the instability proceeds to an almost selfsimilar collapse stage. In this collapse short-wavelength electrostatic fields are generated. These fields are heavily Landau damped, and the inclusion of damping thus results in a rapid loss of the electrostatic energy from the soliton, and finally its disintegration.

In the second sequence of studies planar solitons are generated by an external pump field from initial noise. For 1-d we have compared the results of particle simulations with those from the model equations. These studies have been extended to 2-d using the fluid equations only. We observe the formation and collapse of solitons from initial noise fluctuations. These computations verify the flow of energy from long wavelength pump fields to shorter wavelengths, where it is transferred to the electrons.

(1) V.E. Zakharov, Sov. Phys. JETP 35, 908 (1972)
(2) N.R. Pereira, R.N. Sudan, and J. Denavit, (in preparation)

*Work supported by USERDA under contract E(11-1)3170.

HYBRID IMPLOSION SIMULATIONS OF HEATING IN HIGH VOLTAGE THETA PINCHES*

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The one-dimensional hybrid (Vlasov ion fluid electron) simulation code,[1] including all nontrivial field components and three-velocity components, has been extended and its predictions have been compared with experiment. The effects of incomplete preionization have been included by allowing the simulation ions to be either neutral or ionized initially, and if neutral to be ionized at the rate given by atomic physics as the implosion proceeds and the electrons heat. The resistivity described in Ref. 1 has been satisfactory so far. We present comparisons of theory and experiment for the Los Alamos Staged θ Pinch and IHX experiments, both of which have about 60% preionization, and also for the Scylla 1B experiment, which has 15% preionization. The theory is shown to agree well with experiment in terms of density profiles, magnetic field profiles, and (when available) temperature profiles. In particular, the dominant ion heating mechanism in these experiments is shown to be bouncing off of the imploding magnetic piston. However, we show that the simple analytic nonresistive bounce model predicts grossly incorrect ion dynamics. The effects on the ion dynamics during implosion of variations in the initial preionization level is illustrated. A version of the code coupled to a fairly complex external circuit of the Staged θ Pinch has reproduced the correct boundary conditions for over 6 μs.

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[1]A. G. Sgro & C. W. Nielson, Phys Fluids 19, 126 (1976).

*Work performed under the auspices of the USERDA.

Analysis of a Helical Launching Structure for Alfvén Wave Heating in the Proto-Cleo Stellarator*

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In previous work on Alfvén wave heating,[1,2] the wave launching structure was represented by a current flowing on the surface of a cylinder of radius r_c. To launch the desired mode, a helix is used. The current is represented as a delta function on the surface $r=r_c$. In the azimuthal, θ, direction, these will be an infinite train of delta functions separated by $2\pi/\ell$ radians, where ℓ is the periodicity of the helix in the θ-direction. Defining a toroidal angle $\phi=z/R_o$, where R_o is the major radius of the torus, these will also be an infinite train of delta functions in the ϕ-direction with separation $2\pi/\ell m$, where m is determined by the winding law of the helix, $\theta=m\phi$. Since a sum of delta functions may be represented as a sum of exponentials, the surface current is

$$\vec{J}_s = A\delta(r-r_c)\sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} e^{in\ell\theta}e^{ip\ell m\phi}(\hat{z}\cos\alpha+\hat{\theta}\sin\alpha)e^{-i\omega t} \quad (1)$$

where α is the pitch angle of the helix with respect to the z-axis. Once the helix is wound, both ℓ and m are specified. Modes of wavelength greater than or less than the major circumference of the torus can be launched, depending on ℓ and m (m need not be an integer), n and p. For Proto-Cleo a wavelength along a field line of three times around the torus is required (q=3). Thus m should be 1/3.

However, mechanical constraints required m=7/3. One way to excite the wave is to make the current amplitude vary along the helix. This is imposed by shielding portions of the helix so that it only radiates over discrete segments. Letting the pulse train be represented by a Fourier series, the current is then

$$\vec{J}_s = \delta(r-r_c)\sum_{s=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\sum_{p=-\infty}^{\infty} F_s\, e^{is\gamma(\eta\theta+\xi\phi)}e^{i(n\ell\theta+p\ell m\phi)}(\hat{z}\cos\alpha+\hat{\theta}\sin\alpha)e^{-i\omega t} \quad (2)$$

where γ is determined by the current distribution and η and ξ depend upon the pitch of the helix. The required mode then appears when n=p=0 and the current is a train of 4 positive and 4 negative pulses. The plasma impedance for this mode, as well as for adjacent modes, can then be computed.

[1] J.A. Tataronis and W. Grossman, Proc. 2nd Topical Conf. on RF Heating of Plasmas, Lubbock (1974), paper A6.
[2] A. Hasegawa and L. Chen, Phys. Rev. Lett. 32, 454 (1974).

*Work supported by NSF under grant ENG75-11168 and ERDA contract E(11-1)3077.

Plasma Heating Near the Ion-Plasma Frequency.* J. L. Sperling and C. Chu, General Atomic Company--The response of a magnetized plasma to a pump wave whose frequency lies near the ion-plasma frequency is studied using both analytic techniques and a $1\frac{2}{2}$ D particle simulation. It is shown that the dominant heating mechanism results from an induced scattering of the pump wave by the resonant particles into lower hybrid waves. The resonant wave-particle interactions include electron and ion Landau and ion cyclotron damping associated with a new type of forced plasma response (i.e., quasimode) which exists only in the presence of the pump. The scattering by the electrons results in strong heating parallel to the magnetic field. The induced scattering due to the ion Landau and ion cyclotron resonances results in direct ion heating perpendicular to the magnetic field. In particular, ion Landau and ion cyclotron heating are important when the growth rate of the unstable secondary wave lies above or below the ion cyclotron frequency, respectively.

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Poster Session 1B

Trapped-Particle Modes and Turbulance Theory

NUMERICAL STUDY OF DRIFT-KINETIC EVOLUTION
OF COLLISIONAL PLASMAS IN TORI*

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Preliminary numerical results for the dynamics of toroidally confined plasmas in the drift-kinetic, Fokker-Planck description are discussed. These solutions were obtained by using the techniques inherent to the collisional plasma model (CPM). An initial value problem is solved in the local approximation, collisions and particle dynamics competing in a given magnetic field to set up a quasi-equilibrium. Both the plasma (guiding center) distribution function and many macroscopic quantities of interest are monitored. Good agreement with corresponding but more approximate theories is obtained over a wide range of collisionality, particularly, the neoclassical particle flux. Encouraging confirmation of earlier results for the distribution function is achieved when due account is taken of the differing collisionality for particles of differing energies. These initial results indicate the potential importance of certain non-local effects as well as inclusion of self-consistency between fields and plasma currents and densities.

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+Oak Ridge Associated Universities - Univ. of Michigan

COLLISIONAL EFFECTS ON TRAPPED ELECTRON INSTABILITIES*

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Collisions influence trapped electrons by two basic mechanisms. The first effect is an inability to trap low energy electrons because of collisional scattering. This process determines the shape of the effective boundary between the trapped and untrapped electrons and results in a strongly reduced non-adiabatic trapped electron response for large collisionalities ($\nu_* > 1$). The collisionality $\nu_* = \nu_e/\varepsilon\omega_{be}$; with ν_e and ω_{be} the electron deflection and bounce frequencies for thermal electrons, and ε the inverse aspect ratio.

The second mechanism is the collisional broadening of the untrapped resonant electron response; the trapped electrons being unable to resonant with the wave. The modifications of the non-adiabatic untrapped electron response due to this resonance broadening is analytically evaluated for sheared magnetic fields by employing a pitch-angle scattering collision operator. The results are quite sensitive to the many parameters involved, but there are three general conclusions. 1) For $\nu_* < 1$ the depletion in the number of electrons able to resonant with the wave caused by the presence of a substantial trapped electron population is an important modification. 2) Collisional broadening plays a significant role for $\nu_e/3\omega > 1$ when $\nu_* < 1$, and for $\nu_e/3\varepsilon\omega > 1$ when $\nu_* < 1$, where ω is the wave frequency. 3) In the small ν_* limit the trapped electron effects dominate the resonant electron modifications.

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NUMERICAL STUDY OF THE DISSIPATIVE TRAPPED ELECTRON MODE IN TOKAMAKS

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We have solved the linear dispersion relation for the dissipative trapped electron mode for experimental profiles in four recent tokamaks. Since the full treatment of the two dimensional mode structure is still incomplete, we have suppressed the θ structure to examine the shear induced damping, and also have suppressed the r structure to examine the field aligned mode structure. We find that most tokamaks studied are unstable principally because the destabilizing effect of drift resonances is more important than the stabilizing effect of shear. For $k\rho_i \geqslant 1$, ion-ion collisions are an important stabilizing effect, although this expected to be less important as higher ion temperatures are reached.

In our studies of field aligned structure, we found that parallel ion ineritia and electron Landau damping can often decouple the Fourrier harmonics which make up the mode. The result is that mode structure is flutelike and does not balloon outward. We are now in the process of incorporating ion drifts into our studies of field aligned structure. Finally we have examined the ubiquitous mode.[1] We found that it is a continuation of the ordinary dissipative trapped electron mode to regions of higher $k\rho_i$.

Work supported by ERDA
*Science Applications Inc., McLean, Virginia
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1. B. Coppi and G. Rewoldt, Phys. Rev. Letters 33, 1329 (1974).

Nonlinear Ion Motion in a Slow Ion Cyclotron Mode*

by

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ABSTRACT

The nonlinear cyclotron resonance of a single ion in a coherent slow ion cyclotron wave is treated analytically for arbitrary polarization and angle of propogation in a homogeneous magnetic field. For parameters of interest, the nonlinear phase space orbits are found to be characterized by the case of parallel propogation. For this case, approximate expressions are obtained for the gyroradius and parallel velocity of the resonant particle. In this approximation, the nonlinear period and adiabatic invariant are found. The treatment is shown to extend to a system with slowly changing parameters. It is shown that cooling of the parallel temperature is a natural feature of ion cyclotron resonance heating, an effect which has been seen in a numerical simulation.[1] Comparison of the nonlinear frequency with the linear ion cyclotron and electron Landau damping rates indicates that the ion cyclotron heating experiment carried out on the ST tokamak[2] was pumping into a nonlinear resonant region, before the wave was finally damped linearly. It is found that nonlinear effects could account for the observed loss of high energy ions.

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2. J. Adam, M. Chance, et al, IAEA-CN-33/A3-2

Nonlinear Saturation of the Dissipative Trapped-Ion Mode by Mode Coupling*

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The nonlinear saturation of the dissipative trapped-ion mode by mode coupling is analyzed. The basic mechanism considered is the coupling, via $\underline{E} \times \underline{B}$ convection, of energy in long wavelength unstable modes to short wavelength modes stabilized by Landau damping due to both circulating and trapped ions. In the usual limit of the mode frequency small relative to the effective electron collision frequency and the ion bounce frequency, a one-dimensional nonlinear partial differential equation for the potential can be derived, as first shown by LaQuey, Mahajan, Rutherford and Tang.[1] The stability and accessibility of the possible equilibria for this equation are examined in detail, both numerically and analytically. The equilibrium emphasized by LaQuey et al is shown to be always unstable. However, a class of nonlinear saturated states which are stable to linear perturbations is found.

Included in the analysis are effects of ion collisions and dispersion due to finite ion banana-width excursions. It is demonstrated that while ion collisions have an essential influence on the linear stability of the mode, they do not fundamentally alter the nonlinear saturation of the instability. Dispersion does not directly affect the energy balance between destabilizing collisions and stabilizing ion Landau damping. However, since this balance is mediated by mode coupling which is inhibited by dispersion, dispersive modes saturate at significantly higher amplitudes.

Coherent, cross-field transport is estimated and the scaling of the results is considered for tokamak parameters (specifically those for the Princeton Large Torus). It is concluded that the anomalous cross-field transport can be much lower than the estimate of Kadomtsev and Pogutse for relevant parameters.

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1 R.E. LaQuey, S.M. Mahajan, P.H. Rutherford, and W.M. Tang, Phys. Rev. Lett. 34, 391 (1975)

Trapped-Electron Mode Driven Unstable by Neutral Beam Injection in Tokamaks

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A new branch of the trapped-electron mode, driven unstable by the energetic beam ion-cyclotron resonance $\omega_k - k_{\parallel} V_{\parallel} - k_i V_D - n\Omega = 0$, including the curvature drift $V_D = V_{\parallel}^2/\Omega R$, is found. The growth rate of this mode is calculated and the stability threshold computed for parameters of present injection experiments and the proposed TFTR device. Instability can occur for injection parallel to the magnetic field when the fast ion beam density gradient is directed opposite to the electron density gradient, or for antiparallel injection when the fast ion beam density gradient and the electron density gradient are parallel. The pitch-angle diffusion caused by the trapped-electron mode is calculated from quasilinear theory. Using the free energy available from relaxing the density gradient as an upper bound on the level of fluctuation of the electric field, we find that the quasilinear diffusion coefficient can significantly exceed the classical collisional diffusion coefficient.

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MODELING OF TURBULENT THETA PINCHES
WITH A HYBRID CODE

by

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We have developed a hybrid numerical model of cylindrical theta pinches, based on the self-consistent inclusion of turbulence effects. The effects of various cross-field plasma instabilities upon the electrons are modeled using self-consistent transport coefficients while the effects upon the ions are modeled by appropriate ion heating. This treatment of the turbulent ion heating is the unique feature of our hybrid code which is essential to simulate accurately the dynamics of the reflected ions. As the magnetic piston moves through the plasma and reflects some or all of the ions, the individual trajectories of the reflected ions strongly depend on their azimuthal velocity. Only ions with small angular momentum will reach the cylindrical center. Therefore large azimuthal velocity shortens the path length which a reflected ion must travel before it returns for multiple reflections. Thus the proper computation of the ion turbulent heating near the piston (especially the increase in the azimuthal velocity spread) is important to determine the thermalization rate of the reflected ions and to investigate the eventual equilibrium state of the plasma. Using our hybrid code, we have simulated the Los Alamos Implosion Heating Experiment (IHX) and have obtained numerical results which show very strikingly the mechanisms involved in the experimentally observed plasma motion.

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Impurity Acoustic Instability in the Trapped Electron Regime

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Abstract

The Ohmic current in tokamaks can destabilize the heavy-ion acoustic mode, in which the heavy ions (impurities) provide the hydrodynamic response, with the light ions and electrons as adiabatic species. It is assumed that the impurities are in the Pfirsch-Schluter regime, the light ions are in the plateau regime, the electrons are in the banana-to-plateau regime, and the wavelength of the mode is shorter than the torus connection length. In the absence of electron trapping (plateau regime) a calculation of the electron and ion drift velocities shows that the mode which propagates in the same direction as the ion drift is stable, for all plasma parameter values: the electron Landau damping always slightly exceeds the ion Landau growth. However, when significant numbers of electrons become magnetically trapped (transition to the banana regime) the ion Landau growth can exceed the electron Landau damping (which is inhibited by trapping) and can also exceed the collisional damping by the impurities. Thus, the instability sets in as the electron collisionality ν_e^* is reduced. [The ordinary ion acoustic mode is stabilized by reducing ν_e^*, an effect which has been experimentally verified by Arunasalam, et.al. [1]] Detailed results of calculations will be presented.

1. V. Arunasalam, M. Okabayashi, R.J. Hawryluk, and S. Suckewer, to be published.

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Spectral Distribution of TE-Mode Fluctuations Predicted by a 3D Mode Coupling Analysis*

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The nonlinear stabilization of the drift modes introduced by the divergence of the convective $E \times B$ flux in the ion fluid continuity equation is examined. There are two physically distinct nonlinear effects introduced by this flux. The one effect arises from the Burgers or Korteweg-de Vries type of nonlinearity that produces a steepening of the waveform in its dependence on the phase variable. The magnitude of this waveform distortion is readily calculated at sufficiently small amplitudes, and we conclude that the steepening is a subdominant effect at the saturation levels obtained. The second effect is that of the diffusive or viscous damping in a wave of given $\underset{\sim}{k}\omega_{\underset{\sim}{k}}$ induced by the presence of the other waves. The nonlinear diffusive effect derived here for the localized three dimensional eigenmodes is found to be more complicated than indicated by previous theories that shift $\omega_{\underset{\sim}{k}}$ to $\omega_{\underset{\sim}{k}} + ik_\perp^2 D$ with D determined by the usual consistency relation.

An approximate solution for the stationary spectrum of electron density fluctuations is obtained from the mode coupling equation and evaluated in detail for the microwave scattering experiment performed by Mazzucato[1] on the ATC. In this application of the theory to a plasma in which ν_{*e} is of order unity, the three energy regime approximation for the oscillating part of the electron distribution function is required since significant contributions occur from collision dominated low energy electrons. An extension of our earlier formulation of the collision dominated electron response now includes the effect of electron-electron energy scattering on the previously reported sharp energy response obtained from the Lorentz collision theory. The density, temperature gradient and magnetic shear length are calculated from the density and temperature profiles reported by the ATC experimental group.

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* Work supported by U.S. ERDA E-(40-1)-4478.

Scaling Laws for Ion Acoustic Turbulence*

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Abstract

As reported earlier[1] the renormalized theory has been used to develop a mode simulation of ion acoustic turbulence for a constant current. Suprathermal ion production as predicted by the theory and incorporated in the mode simulation gives turbulent pulses with a typical pulse width of 50-100ω_{pi}^{-1}. For electron velocities

$$u < u_{crit}\left(\frac{m_i}{m_e}, \frac{T_e(o)}{T_i(o)}\right) \sim (0.3 - 0.5)\, v_e(o)$$

the mode simulation shows that the fraction of suprathermal ions at the end of the turbulent pulse $n_t(\tau_f) \propto u/v_e(o)$ and that at the pulse maximum $\nu_{eff}/\omega_{pe} \simeq 1.2 \times 10^{-3} \frac{u}{v_e(o)} \frac{T_e(o)}{T_i(o)}$ for $m_i/m_e = 1600$. For $u > u_{crit}$, $\nu_{eff}/\omega_{pe} \leq .02$ at the pulse maximum and $n_t(\tau_f) \sim .16$, independent of $u/v_e(o)$, while $T_e(\tau_f)/T_e(o) \simeq 50\,(u/v_e(o))^2$ for $m_i/m_e = 1600$. These results are explained in terms of the behavior of the average mode. The results of these studies are also compared with 2D particle simulations of ion-acoustic turbulence where results are available for a subset of the parameters considered here. Similarities and differences in the heating and turbulence levels observed in particle and mode simulations are reported.

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Theory of the Nonlinear Saturation and Anomalous Transport due to Drift Instabilities*

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Low frequency microinstabilities in a magnetically confined plasma have attracted increasing attention due to the important roles they may play in the anomalous transport of the plasma. The existing linear theories have predicted the onset conditions of these instabilities, yet the saturation spectrum which determines the confinement time of the plasma can only be obtained from the nonlinear theory. Several nonlinear saturation mechanisms have been proposed, however there is no detailed comparison with the experimental results so far.

It is the purpose of this paper to present a self-consistent theory which accurately describes the nonlinear evolution of the instability and demonstrates the saturation of the waves through the flattening of the background plasma density profile. The theoretical predictions agree very well with the recent simulation results[1]. In particular, anomalous heat transfer due to the local absorption of the wave energy through Landau damping has been confirmed theoretically.

The outline of the theory is the following. For a given density and temperature profiles of the plasma, the eigenvalues for the frequency and amplitude of the waves are determined from the two coupled normal mode equations derived from a multiple-time-scale expansion. The frequency and the amplitude are then used in the macroscopic transport equations to evaluate the particle diffusion and the energy transport due to the waves. It should be emphasized that these transport equations are derived from the Vlasov equation taking into account the Landau absorption and emmision of the wave energy essential to heat transfer. The resulting density and temperature profiles are then substituted back to the normal mode equation again, and the same procedures are repeated to follow the time evolution.

The theoretical results are compared in detail with those from the simulation and have confirmed (1) dominant saturation mechanism is the flattening of the density gradient, and (2) energy diffusion is much larger than the particle diffusion due to the local absorption and emmision of the wave energy. The latter may be related to the anomalous heat loss in tokamaks.

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Numerical Simulation of a Turbulent Heated Plasma for a 'High Beta Tokamak'*

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A proposed turbulent heating scheme by a fast reversal of the toroidal field for Columbia Torus II is demonstrated by numerical simulation. The plasma motion, Ohmic heating, and field diffusion are studied by our two-dimensional code, in which the anomalous conductivity is varied as const/j (j is the current density in the plasma), where the constant varies between Buneman and classical values. The results show that a dense plasma can be heated rapidly up to several hundred electronvolts without pinching. For the sake of calibrating the numerical code, we also simulate the TEE machine of Julich and get good agreement with the experiment on the temperature and beta value.

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MARGINAL STABILITY CALCULATION OF TRAPPED ELECTRON MODE ANOMALOUS TRANSPORT IN TOKAMAKS

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We hypothesize that the dissipative trapped electron instability causes anomalous transport sufficient to drive the plasma profile to a state where the mode will be near marginal stability, and calculate the consequences of this assumption. The results of our calculations indicate the reasonableness of this approach. By balancing the shear induced wave damping with the dissipative trapped electron wave growth, we obtain a differential equation relating $q(r)$ and $T_e(r)$. This, together with the curl $\underset{\sim}{B}$ Maxwell equation, allows a direct calculation of the electron temperature profile, $T_e(r)/T_e(o)$, and the wavelength of the mode. The energy equation then allows us to calculate the anomalous electron thermal transport $Q_e(r)$ necessary to produce this profile. Quasilinear theory (which relates $Q_e(r)$ to the fluctuation level) then allows us to determine $(e\phi(r)/T_e)^2$ for the mode. As an example, we calculate confinement times and radial profiles of $T_e(r)$, $q(r)$, $\langle(e\phi/T_e)^2\rangle$, fluctuation mode number, heat and particle flux, and resistive anomaly for parameters typical of the PLT device. An important result is that knowledge of the classical energy loss mechanisms near the plasma edge (e.g. loss to the limiter and radiation) are crucial in determining the central temperature and energy confinement time.

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Fluid-Like Instabilities Driven by the Magnetic Curvature Drift*

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In high temperature magnetically confined toroidal plasmas, the unfavorable magnetic curvature seen in the average by trapped electrons and, locally in the outer part of the torus, by all ions, can drive a fluid-like [1,2] instability with the amplitude of the relevant electrostatic potential along a magnetic field line, even around the point of minimum magnetic field.

A similar instability, with the amplitude of the electrostatic potential localized in a region around the point of minimum magnetic field but odd along a magnetic field line, can be driven by the magnetic curvature drift of ions alone.

In both cases the spectrum of the relevant perpendicular wavelengths $\lambda_\perp$ extends up to values larger than the ion gyroradius when realistic estimates of the torus aspect ratio are made. This portion of the spectrum is shown to give a significant contribution to the energy and particle transport across the magnetic field.

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2. B. Coppi and G. Rewoldt, Phys. Lett. 49A (1974) 36

* Work supported in part by the U.S. Energy Research and Development Administration

Poster session

Two-Dimensional Spatial Structure of the Dissipative Trapped-Electron Mode*

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ABSTRACT

Previous investigations of the spatial structure of the dissipative trapped-electron mode have generally been one-dimensional. In particular, either the radial structure of the mode is ignored while solving for the structure parallel to the magnetic field line,[1] or the parallel structure is ignored while solving for the radial structure in the vicinity of a single mode-rational surface.[2,3] This paper deals with the complete two-dimensional structure of the mode over its full width, which may extend over several mode-rational surfaces. The complete integro-differential equation is studied in the limit $k_\perp\rho_i < 1$ (where $k_\perp$ is regarded as a differential operator) for the case of a circular cross-section, large aspect ratio tokamak with concentric magnetic surfaces. The perturbed electrostatic potential is expanded in complete sets of radial and poloidal basis functions, thereby converting the integro-differential equation into a matrix equation, for which eigenvalues and eigenvectors can be obtained by standard numerical methods. Periodicity requirements in the presence of magnetic shear are satisfied, and the effects of temperature gradients, electron and ion magnetic drifts, electron (Krook) collisions, and ion resonances are included in the analysis. However, resonant electron effects have not yet been added. Solutions obtained are in reasonably good agreement with the one-dimensional analytic solutions referred to above, in the limits where such results are expected to be valid. More significantly, the present approach can readily treat many physically important cases for which purely analytic solutions are difficult to obtain.

*Work supported by U.S.E.R.D.A. Contract E(11-1)-3073.

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[2] A. A. Galeev, 3rd Int. Symp. on Toroidal Plasma Confinement, Garching (1973)

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<u>Small Amplitude Nonlinear Saturation of Finite, Collisionless Plasma</u>. ALBERT SIMON, <u>U. Rochester</u> and M.N. ROSENBLUTH, <u>Institute for Advanced Study</u>. -- A general technique has been developed which allows us to extend the Bogoliubov-Krylov nonlinear asymptotic analysis to slightly unstable collisionless plasma. The method is illustrated by application to the bump-on-tail instability of a bounded plasma, where a single mode is made unstable by a fractional density increase, Δ. We demonstrate that all frequencies of higher order in $\Delta^{\frac{1}{2}}$ are generated by self-beating of a basic Case class 1c mode. The Van Kampen modes are shown to have zero-amplitude asymptotically. All singular functions are regularized by a limiting procedure. A saturation equation follows from a self-consistent expansion condition, in addition to the usual non-secularity requirement. The expressions for the limiting amplitude and frequency shift are entirely unlike the usual result of quasilinear theory and depend on several derivatives (up to the fifth) of the distribution function at the bottom of the velocity-distribution well. The final state is shown to be stable, and conserves energy and momentum.

PARTICLE HEATING BY DIFFUSION IN A SINGLE WAVE*

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ABSTRACT

A particle distribution evolves according to a diffusion equation if a broad spectrum of waves is present. In a magnetized plasma, even a very narrow spectrum (modeled as a single wave) can lead to a diffusion equation if the wave amplitude is large enough. If the potential Φ of an electrostatic wave satisfies $e\Phi \gtrsim m(\Omega/k_z)^2$, where $\Omega = eB_0/mc$, particles with gyroradius $\rho \gtrsim k_\perp^{-1}$ move stochastically.[1] Such particles are not trapped near a single resonance $v_z = (\omega + \ell\Omega)/k_z$, but move from one resonance region to another. Diffusion in velocity space occurs along the curves $(v_z - \omega/k_z)^2 + v_\perp^2 =$ const. In the stochastic region of velocity-space $(k_\perp \rho \gtrsim 1)$, we calculate the diffusivity

$$D \equiv \langle [\Delta v_z(t)]^2 \rangle / 2t \sim (0.1)(k_z e\Phi/m)^2/\Omega .$$

There occurs rapid transfer of energy from the wave to particles in the tail of the distribution. We have verified the results of our analysis, by evaluating D from a set of numerically calculated particle trajectories.

* Work supported in part by ERDA.

1. G. R. Smith and A. N. Kaufman, Phys. Rev. Lett. <u>34</u>, 1613 (1975).

Residual and Ion Diamagnetic Trapped-Particle Modes*

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This paper addresses two important questions, namely (1) can residual forms of trapped-particle instabilities persist in systems with flat or reversed gradient profiles? and (2) what conditions are necessary to excite ion diamagnetic trapped-particle modes? The first question is relevant because it has been proposed that in future tokamaks it will be possible to create profiles which can effectively suppress the usual types of trapped-particle instabilities.[1] In the present analysis it is demonstrated that residual modes with generally smaller growth rates can still persist. Regarding the second question, it is shown that trapped-ion in the <u>ion</u> rather than the usual electron diamagnetic direction can readily appear. The residual modes mentioned above exhibit this characteristic. Moreover, for sufficiently large ion temperature gradients, the ion diamagnetic branch is found to be dominant even for normal density profiles. This branch can be destabilized by resonances with ions that have average unfavorable curvature drifts. For trapped-electron modes conditions for the onset of instabilities in the ion diamagnetic direction ("ubiquitous" modes)[2] are derived for relevant tokamak operating regimes.

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[1] W.M. Tang, et al., Phys. Rev. Lett. <u>35</u>, 660(1975).

[2] B. Coppi and G. Rewoldt, Phys. Rev. Lett. <u>33</u>, 1329(1974).

NUMERICAL STUDIES OF NEOCLASSICAL RESISTIVITY IN TOKAMAKS*

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The resistivity of a tokamak plasma is of great interest to controlled thermonuclear research because, at least for the initial phase, a tokamak reactor will rely primarily on ohmic heating. For $\epsilon \neq 0$ and $E_{\parallel}/E_D \ll 1$, the resistivity is larger than classical[1], i.e. $\eta_{\parallel}/\eta_{c\ell} > 1$, where $\epsilon = r/R_o$ is the aspect ratio, $E_{\parallel}$ is the applied electric field parallel to B, $E_D = m_e v_{th}/e\tau_{ei}$ is the Dreicer field, $v_{th} = (T_e/m_e)^{\frac{1}{2}}$, $T_e = \frac{m_e}{3}\int f_e v^2 dv$, $\tau_{ei} = 3.5\times10^5\, T_e^{\frac{3}{2}}/(\ell n \Lambda\, n_e)$, $\eta_{\parallel}$ is the parallel resistivity and $\eta_s = 2m_e/(\tau_{ei} ne^2)$ is the Spitzer resistivity for hydrogen. This is due to the "neoclassical" effects of electron trapping by magnetic wells, and the distortion of magnetic surfaces in tokamak geometry. As $E_{\parallel}/E_D$ increases, it was generally believed that a long runaway tail in $f_e(v)$ would develop and a large hard x-ray flux would be detected. This is indeed observed in some tokamak experiments.[2]

Recently, low density alcator discharges[3] showed, instead of a long runaway tail carrying most of the current, that a large portion ($\lesssim 40\%$) of $f_e(v)$ is "sliding" away with very low level of hard x-rays. Strong ion heating with rf emission at ω_{pi} was observed. Recent theories[4] suggest that a bump might develop in $f_e(v_{\parallel})$, where $f_e(v_{\parallel}) = \int d\underset{\sim}{v}_{\perp} f_e(\underset{\sim}{v})$, which drives unstable modes near ω_{pi} and heats the ions.

This paper summarizes our numerical investigations of the neoclassical resistivity and the structure of the electron distribution function in the neoclassical regime for $0.01 \ll E_{\parallel}/E_D < 1$.

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*Work supported by ERDA, †Science Applications, Inc.

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Poster Session 2B

Exploratory Concepts and General Plasma Theory

FLUTE INSTABILITY OF A LASER-PELLET PLASMA IN A MAGNETIC MIRROR*

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ABSTRACT

One scheme for start-up in magnetic mirror devices involves laser irradiation of a small pellet. The expanding plasma is unstable to Rayleigh-Taylor modes which give rise to fluting which could cause loss of plasma from the mirror if the flutes continue across the field lines. On the other hand a beneficial effect could occur since, in the absence of any instabilities, the maximum volume that would be filled in a device such as Baseball IIT is about 3 cm,[3] too small for use as a target plasma.

We have used plane and cylindrical snowplow models[1] to analytically follow the instabilities into a moderately nonlinear regime. For the most important range of wave numbers, the Lagrangiam equations for the nonlinear evolution of the pellet boundary reduce to a set of linear equations previously derived for a gravitating sheet mass by Ott.[2] These equations have been solved exactly; the solution shows that an initially sinusoidal surface perturbation steepens to form cusps. Other surface perturbations have been studied and also show cusp formation.

The snowplow model breaks down before cusps are formed; to proceed further, we have used a numerical MHD code.[3] With this code we have been able to study the nonlinear evolution of different perturbations of the plasma surface. We have verified that the long wavelengths dominate the behavior at long times. As the bulk of the plasma decelerates, the flutes continue into the magnetic field, their front surface forming a mushroom shape. This front surface is in turn unstable and smaller flutes form which are eventually stopped by the field.

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Cyclotron Radiation in Fusion Experiments

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Cyclotron radiation is important (1) as an energy loss mechanism, especially if advanced fuels - and hence higher temperatures and electron densities - are envisaged (2) as an internal energy transport mechanism, influencing the temperature distribution inside the plasma, (3) as a diagnostic tool.

The optical constants of a plasma have been calculated at the harmonics of the electron cyclotron frequency, including higher-order density effects. The theory, which is an extension of previous work where ω_p^2/ω^2 was assumed negligible with respect to unity, has no restrictions with regard to the direction of propagation or the nature of the velocity distribution function. In its present form, electron temperatures above 10 keV are excluded, although this temperature range can easily be extended.

The theory has been applied to find the radiative energy load at the first wall of a tokamak, and to explain the cyclotron emission spectra observed on the TFR-device. In the latter case the agreement is reasonable to good. A simplified model was used which gives good results only if the ray which reaches the observer has passed through the center of the plasma during its last traverse. This requirement is often fulfilled in diagnostic measurements, as in many cases the line of observation lies in the equatorial plane of the torus.

In both cases, the strongest harmonics are those at which the plasma is in the transition region between optically thin and optically thick, which is an indication of the importance of the reflectivity of the walls.

In a toroidal machine with small aspect ratio (e.g. a "fat" tokamak) and for temperatures of several keV, the overlapping between harmonics becomes so strong that separate spectral lines can no longer be resolved. As long as the plasma is optically thick, the intensity increases with the square of the frequency. The spectrum shows a flat maximum at the first several harmonics where the plasma is no longer optically thick, and in the optically thin region decreases with a $(v_{th}^2/c^2)^{m-1}$ law, m being the harmonics number.

For lower temperatures, such as in the TFR-device, the plasma is thick at the second harmonic, and at the third it is in the intermediate region, its intensity there being 80% of that at the second. At the fourth harmonic, which is hardly vsible, it is already optically thin.

Stability of the High Energy Runaway Electron Component in a Magnetized Plasma*

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The stability of the high frequency electrostatic modes of the magnetized plasma in the presence of the high energy electron component is analyzed. A statistical particle simulation code capable of describing the pitch angle scattering, the drag, and diffusion in the presence of an applied external electric field is used to follow the time development of the high energy electron component. The stability of the resulting electron distribution function with respect to the magnetized Langmuir mode and the electron cyclotron mode is monitored locally in time. When the instabilities are triggered the added quasilinear contribution from the suprathermal fluctuations is incorporated in the simulation code. In this regime the turbulent and classical processes are treated on the same footing. Analytical results for thresholds of the instabilities are reported and the relevance of the present work in connection with the T-6 experiment[1] where the various high frequency instabilities are observed in the runaway discharge is discussed.

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[1] V.S. Vlasenkov, et al., Nucl. Fusion 13, 509 (1973).

* The work was supported by the U.S. Energy Research and Development Administration Contract E-(40-1)-4478.

On the Effects of a Hall Current in Magnetohydrodynamics

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It has been proposed in the past that a Hall current may have important implications regarding MHD instability[1] and wave phenomena[2]. We examine this conjecture by studying the ideal MHD model with the Ohm's law replaced by $\underset{\sim}{E} + \underset{\sim}{v} \times \underset{\sim}{B} = (\varepsilon/\rho)\underset{\sim}{J} \times \underset{\sim}{B}$, ε and ρ being respectively, the charge to mass ratio and the mass density of the ions. Among the questions that we shall address ourselves to are the general properties of the proposed system of equations, the relevant boundary and jump conditions that must be satisfied in this model, the spectrum of the linearized equations about inhomogeneous equilibria with electron and ion flow, and finally pathologies of the model. The principle conclusions are: (i) the MHD equations with a Hall current are non-hyperbolic and dispersive; (ii) the Alfvén and slow waves are absent in the model, while the fast wave degenerates into an acoustic wave; (iii) the ideal MHD continuous spectra associated with the Alfvén and slow waves do not exist, but continuous spectra of the Couette type resulting from the ion and electron flow appear; (iv) questions regarding the well-posedness of the linearized initial value problem in the Hall current model appear due to the existence of frequency independent singularities at points in space where $\underset{\sim}{k} \cdot \underset{\sim}{B} = 0$, $\underset{\sim}{k}$ being the effective wave vector of the perturbations.

(1) Ware, A.A., J. Nucl. Energy, Part C, 3 (1961)93
(2) Kadish, A., Physics of Fluids, 19, 141(1976)

The Potential of Multipoles as Thermonuclear Reactors

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Recently detailed experiments on plasma confinement in Torroidal Octapoles have been carried out by the Wisconsin group. The conditions for these experiments were $n_e \approx 5 \times 10^9$, $T_e \approx 5$ev, $T_i \approx 10$-20ev. It was found that for levitated rings and poloidal field only the diffusion coefficient scaled like $D \approx 100\, V_{Ti}(\ell n k_{MAX} r)^{\frac{1}{2}}(n\ell_m)^{-\frac{1}{2}}$ where ℓ is the length of a closed line of force and r is roughly the minor radius of the device. The introduction of supports in this case made little difference. If a modest torroidal field was introduced the diffusion rate dropped by an order of magnitude and appears to scale according to the law $D \propto \sqrt{T}/n$. If supports were introduced in this case, the diffusion rate increased by an order of magnitude. In both cases the diffusion rate was independent of B.

If we assume that these scaling laws will hold up to thermonuclear temperatures and densities we may estimate how large a device must be to make a successful reactor. For the case of floating rings a size about 10 times the existing octapole is required and the size is determined by the skin time for the rings rather than plasma diffusion. Operation in this mode requires a method of periodically cooling the rings. With supports the plasma loss rate is larger but it appears that it should still be possible to build a reactor of this type which is comparable in size to a tokamak reactor.

The good confinement and low synchrotron emission from such devices makes them attractive candidates for use with exotic fuels such as pB''. Because of the lack of neutron production one can consider using superconducting rings. Operation with such fuels will be difficult but the potential advantages are such as to warrant serious consideration. Some discussion of this will be given.

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Plasma Oscillations of Bounded Magnetized Plasma

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Surface waves in a homogeneous plasma slab bounded by vacuum and containing an external magnetic field have been simulated with a 2½ D electrostatic particle code. In this model, particles are reflected from the boundaries. The case where the magnetic field is parallel to the boundary has been investigated. For electron waves propagating along B, there are symmetric and anti-symmetric modes with frequencies between the cyclotron frequency Ω_e and the plasma frequency ω_{pe}. Surface waves propagating across B no longer have the same symmetry. Rather the modes correspond to rotation with and counter to the cyclotron gyration, and have frequencies between zero and the upper hybrid frequency ω_{UH}.

The dispersion relation based on a warm fluid model with cold ions has been derived and agreement with those frequencies is good. Additional frequencies not predicted by fluid theory have been found in the simulation for waves propagating across B. They have a dispersion reminiscent of Bernstein waves, but there exists no analytical theory for these modes. In each case the surface waves were found to be strongly damped.

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PELLET PLASMA INTERACTION THEORY*

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A theory of the behavior of solid hydrogen pellets in thermonuclear plasma is described in which a high density neutral gas layer surrounding the pellet alternately shields the unablated pellet, and then disperses by expansion.

We calculate deposition profiles based on this and a simpler, steady-state model. We examine the scaling of pellet penetration depths and lifetimes in tokamak plasmas and estimate the resultant transport of neutrals due to the partial ionization of the ablatant.

*Research sponsored by the U.S. Energy Research and Development Administration under contract with Union Carbide Corporation.

The Free Electron Laser

T. Kwan, J.M. Dawson and A.T. Lin

Experiments at Stanford[1] showed that a relativistic electron beam passing over a rippled magnetic field can amplify light waves of 10.6μ with 7% gain per pass. Also, generation of large amounts of microwave power has been detected in similar experiments at NRL[2]. If the ripple wavelength is λ_0, the lasing wavelength is roughly $\lambda_0/2\gamma^2$. Thus such a laser is continuously timable by varying γ. We have carried out investigation of this possibility for a new type of laser by computer simulation using a $1^2/_2$ dimensional (1r coordinate, 3 velocity coordinates) fully relativistic electromagnetic particle code. Electromagnetic radiation with frequencies up to 14 ω_{pe} has been observed. We also find that as much as 35% of the beam energy can be converted into radiation, of which as much as 90% can be in the most rapidly growing mode. A theory of the coupling between the negative energy plasma wave and the electromagnetic wave through the rippled magnetic field is formulated. We have obtained good agreements between the simulation and the theory. The saturation mechanisms is found to be the trapping of the beam by the unstable plasma wave. We are also able to estimate theoretically the amounts of energy that can be converted into radiation from the electron beam.

Work supported by NSF, Contract PHY74-15233

[1]Science 187, No. 4181, March 21, 1975. Research News; Physics Today, February 1976.

[2]M. Friedman and M. Herndon, Appl. Phys. Lett. 22, 658 (1973).

Very Low Threshold Finite β Streaming Instabilities

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We have extended and solved our previous equations[1-3] for the plasma dispersion relation for a Maxwellian plasma with currents streaming along the magnetic field to include finite $T_\perp$. The equations allow for arbitrary, wave vector $\vec{k}$, ω_p/Ω_-, β and ion-electron temperature ratio (ω_p = electron plasma frequency). The only restrictions are that the density and magnetic field be uniform. We have concentrated on the case appropriate to the TCT or CBT, i.e. stationary electrons and ions streams with equal and opposite velocities, V_D. There are four instabilities in the range $\tilde{V}_+ \approx 5$ ($\tilde{V}_+ = V_D / \sqrt{2T/m_+}$) . Three of these, the lower hybrid mode, the purely growing nearly electrostatic (ES) mode and the ion-cyclotron mode, all with $\vec{k}$ vectors nearly perpendicular to the magnetic field $\vec{B}_0$ have been discussed by us[3] for the $T_\perp = 0$ case. The fourth mode has $\vec{k} \,||\, \vec{B}_0$ and is evidently an electromagnetic (EM) Alfven or hose type mode. A typical result obtained through the Queens College Sigma 7 time sharing computer for the case where all the ions stream, and all temperatures are equal is that instability exists for $\tilde{V}_+ > 1.6$ (β = .1), for $\tilde{V}_+ > 4.0$ (β = .01) and for $\tilde{V}_+ > 6.2$ (β = .001) for the case that $\omega_p \gg \Omega_-$ and $\beta = 4\pi n_e T/B_0^2$. (The results are quite similar when $\omega_p \approx \Omega_-$). The most sensitive instabilities are the $\omega_r = 0$ modes with growth rates $\approx .2\Omega_+$. The ion-cyclotron modes ($\omega_r \neq 0$), are negligible . We note the surprising sensitivity of the equations to the value of β . We give the detailed parameter dependences of the growth rate on the beam strength and temperature of the various constituents.

1. R. W. Landau, J. Plasma Phys. 13, 377 (1975)
2. R. W. Landau, Phys. Rev. A 12, 2168 (1975)
3. R. W. Landau, MATT - 1135, July 1975, Princeton Lab report

Nonlinear Dynamics of Runaway Electrons and Enhanced Synchrotron Emissions from Tokamaks

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The Fokker-Planck equation for Coulomb collision with a driving electric field is solved in the runaway region to obtain a steady state distribution function. This distribution is found to be unstable to the electrostatic waves with two kinds of instabilities: Type (1) is driven by the anisotropy of the runaway electrons with the anomalous Doppler resonance $\omega - k_{\parallel} V_{\parallel} = -\Omega_e$ requiring a density ratio of the runaway to the bulk plasma, Δ, exceeding 10^{-3} to overcome the background damping. Type (2) is driven by the bump in tail of the runaways where $\partial f/\partial v_{\parallel} > 0$ [i.e., at $V_{\parallel}/V_{th} > 6.4\ (E_D/E)^{1/2}$, $V_{th} = (2T/m)^{1/2}$, $E_D = \frac{4\pi n e^3}{T} \ell n\Lambda$. The stability boundary is determined. The quasilinear equation is solved analytically to obtain the time evolution of the runaway distribution function which in the limit $\omega_p \ll \Omega_c = eB/mc$ is

$$f_R(V_{\parallel}, V_{\perp}, \tau) = \frac{\Delta \exp[-(V_{\perp}^2/V_{th}^2)(\lambda/4\tau\lambda+1)]}{\pi^{3/2} v_{max} v_{th}^2 \ell n(EV_{\parallel}^2/E_D v_{th}^2)(4\lambda\tau+1)}$$

where $\lambda = E/E_D \ell n(EV_{\parallel}^2/E_D V_{th}^2)$ and

$$\tau = \frac{\pi}{2}\omega_p^2 \int_0^t dt' \int d\vec{k}\, \frac{|E_k|^2(t')}{8\pi nkT} \frac{k_{\perp}^2}{k^2} \delta(\omega_k - k_{\parallel} v_{\parallel} + \Omega) .$$

Two saturation mechanisms are considered (i) the quasilinear stabilization and the subsequent damping and (ii) the convective stabilization, with the former dominant in large Tokamaks while the latter dominant in present machines. The energy transport by enhanced waves is calculated. The enhanced synchrotron emission from this runaway distribution is also computed and compared with experiments.*

*A. W. Trivelpiece, D. A. Boyd, C. M. Celata, F. J. Stauffer, private communication.

CALCULATION OF $(\overline{\sigma v})_{DT}$ FOR

ANISOTROPIC MIRROR AND TOROIDAL DISTRIBUTIONS*

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An accurate and very fast method is developed to compute the reaction rate parameter

$$(\overline{\sigma v})_{DT} = \frac{2\pi}{n_D n_T} \int_0^{\infty} v^2 dv \int_0^{\infty} v'^2 dv' \int_0^{\pi} \sin\theta d\theta \int_0^{\pi} \sin\theta' d\theta'\ f_D(v,\theta)\, f_T(v',\theta') \cdot \int_0^{2\pi} d\phi\, \sigma(u)\, u$$

where n_D and n_T are the deuterium and tritium densitites, σ is the reaction cross section, u is the relative velocity between species, and f_D and f_T are the deuterium and tritium distribution functions. The 5-D integral is determined by expanding both the distribution functions and the integral over ϕ in Legendre series. This procedure is advantageous because once an initialization procedure is completed, $(\overline{\sigma v})_{DT}$ may be computed inexpensively, and therefore frequently. Also, this method can be conveniently adapted to existing Fokker-Planck codes in which Legendre expansions of the distribution functions are already computed to facilitate evaluation of the collision operator.

The distribution functions which are employed are representative models appropriate both to toroidal devices with counterstreaming ions and also to mirror machines. A 3-D integral, which is an exact approximation to the 5-D integral in the event that one of the species is isotropic, is employed for comparison purposes. It is found that the error in the 3-D integral is at most 8% for mirror distributions, where the minimum mirror ratio studied was 3. However, the error for counterstreaming ion distributions is generally much greater, on the order of 35% at an average energy of about 30 KeV. One can conclude that it is relatively important to utilize this 5-D approach for counterstreaming toroidal distributions, but somewhat less so for mirror distributions.

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Adiabatic Invariance
of the Nonlinear Hamiltonian

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The slow modulation of an arbitrarily nonlinear dynamic system of one degree of freedom is examined. The action is proved to be 'invariant' to exponential precision, if the Hamiltonian is analytic.

Monte Carlo Calculations on a Method of Enhancing Mirror Confinement*

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Experiments with the LLL 2XIIB mirror machine have demonstrated high beta plasmas, achieved by pulsed neutral beams, in which a localized deep magnetic well is created by the rapid buildup of trapped hot plasma. In principle the length of this well could be made to increase with time through control of the time history of the angular and spatial distribution of the neutral beam sources. This technique might therefore be used to create, transiently, a mirror confinement region in a d.c. external field in which the length of the confinement region increases as a function of time during buildup so that trapped ions will reflect from outwardly moving mirrors, a process which takes away parallel energy and tends to reduce the mirror leak rate (which depends on the ratio of parallel to perpendicular energy). This effect could enhance the mean lifetime of the injected particles, as averaged over the buildup, depending on quantitative factors.

To estimate the improvement in confinement that might be achieved for particles trapped between moving mirrors, calculations have been performed using Monte Carlo techniques. In the simplified computational model used the total particle energy remains constant under both ion-ion scattering and reflection from the mirrors; only the pitch angle is changed. The results of the calculations may be adequately expressed as a linear function of two dimensionless variables: $x = \tau_o/\tau_m$, the ratio of the mean particle lifetime, τ_o, when the mirrors are stationary to the time, τ_m, that it takes for the mirrors to double their separation distance and $y = \tau_v/\tau_o$, ratio of enhanced confinement time evaluated while the mirrors are moving apart (with velocity v) to τ_o. For mirror ratio 5, $\theta_{inj} = .2\pi$, $x < 1.0$, $y \approx (1.0 + 2.7\ x)$ i.e. for example, when τ_m is 5 times τ_o ($x = .2$) the confinement is enhanced by about a factor 1.5. Thus in this simplified model a relatively slow separation of the mirrors is sufficient to produce a substantial increase in confinement time, particularly for injection close to the loss cone angle.

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Imaging of Plasma Density Profiles by Computer Reduction of Microwave Diffraction Patterns

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This paper discusses the relative merits of various microwave plasma diagnostic techniques using both spatial and spectral resolution. Numerical data reduction methods which take advantage of cheap high-speed computing facilities to increase accuracy of density calculations are examined.

Plasma diagnostics using microwaves traditionally make use of phase shifts along a line-of-sight path. Refractive effects are usually minimized experimentally, or corrected for in the data reduction. High density plasmas, however, may be usefully characterized in terms of diffraction patterns if the opposite track is taken, i.e. operation near ω_p so that refractive effects are large. Low frequency measurements may be simpler, cheaper, and more informative than multi-beam interferometers at sub-millimeter or micron wavelengths. Disadvantages are that multiple wall reflections will be troublesome, and that interpretation requires complex unintuitive numerical analysis.

The theory of diffraction from a plasma column is discussed, and approximate methods are compared to exact solutions.

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Abstract Submitted

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Subject Heading
Feedback and Dynamic Control

It is known that constant feedback using one sensor and one suppressor fails to stabilize more than one mode of a plasma instability. We show that a suitably frequency dependent feedback can stabilize all modes. We develop a method of formulating such a frequency dependent stabilizing feedback for any general plasma instability. First, using a new concept of dynamical states of a plasma system we derive a state space model of the plasma instability. Secondly, from the single sensor signal we estimate all the states of the problem via a State Reconstructor. Thirdly, using the estimated states from this Reconstructor we employ state feedback to stabilize all modes.

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ELECTRON BOUNCE MODES IN MIRROR PLASMAS*

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ABSTRACT

Electron bounce modes can occur in mirror plasmas when the thermal spread in the bounce frequency is small. This condition is satisfied in mirror devices when electrons are principally confined by an approximate electrostatic harmonic force. These modes are examined by numerically solving an integral equation for the perturbed wave potential in a mirror plasma. A long wavelength mode is found which can be destabilized by ions because of the loss cone nature of their distribution. Threshold densities and maximum growth rates are calculated using a perturbation method. The theoretical stability threshold predictions agree closely with Baseball II measurements and show qualitative similarity with Baseball I data.

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Non-Circular Cross Section Stellarators*

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Several questions need to be answered about the nature of non-circular cross-section stellarators. For example, what is the maximum β that can exist in such a device without destruction of the magnetic surfaces? What are the equilibrium and stability properties of such configurations? What are the forces on the windings? Would advantages similar to those of non-circular cross-section tokamaks also appear? Recent work [1] describing a straight non-circular cross-section stellarator of high poloidal wave number, (ℓ=6), has prompted an investigation, as a first step, of the nature of the magnetic surfaces, flux, and rotational transform in a fully-toroidal non-circular cross-section stellarator to determine the effects of toroidicity on these quantities, especially for low values of poloidal wave number ℓ. Wang and Jensen [2] have examined high poloidal wave number stellarators ($\ell \cong 50$) with an additional quadrupole winding.

The non-circularity is introduced by winding the helical conductors on the surface of an elliptical torus. The major axis of the ellipse is parallel to the major axis of the torus. The nature of these surfaces is partly dependent upon the winding law used for the helical conductors.

An ℓ=2 non-circular cross-section configuration has magnetic surfaces which are elliptical in shape but the general form does not appear to rotate with increasing toroidal angle as does the circular cross-section case. Two stagnation points at the top and bottom of the closed magnetic surfaces appear. The presence of these stagnation points might result in effects on the equilibrium, stability, etc., but such an analysis is beyond the scope of this work. However, divertor action can be expected due to the presence of these stagnation points. For ℓ=3 case a separatrix is seen to produce a stagnation point on the magnetic axis. The rotational transform is smaller than for an equivalently sized circular cross-section stellarator using the same currents. In tokamak devices, the lower rotational transform (higher q) is an advantage, since higher ohmic heating currents can be used to give the same q. However, it is not clear from this initial analysis whether this decrease in rotational transform will be deleterious on the equilibrium and confinement in a stellarator and more work needs to be done to determine whether this configuration will be useful.

[1] I.S. Danilkin and L.M. Kovrizhnykh, JETP Lett. 19 (1974) 119.
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THE HEATING AND CONFINEMENT OF SLENDER HIGH-BETA FUSION PLASMAS RELEVANT TO THE LASER SOLENOID CONCEPT*

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A slender high-beta D-T plasma column contained in a linear solenoid is examined to determine its thermonuclear burn properties and end-loss characteristics. Thermonuclear burn is treated by means of a two temperature time-dependent model based on an idealized radial and axial structure. Integrated quantities are used to simulate the unthermalized alpha particles which are assumed to be independent particles undergoing cyclotron motion while gradually losing energy to electrons and ions. It is concluded that slender plasma columns are heated primarily by strong resistive heating (i.e., plasma-field radial diffusion) below about 7 keV and by alpha particle heating at higher temperature. Large alpha particle Larmor radius excursions outside the plasma reduce the rate of heating. The increase in temperature is terminated eventually by axial electron conduction loss.

Particle end-loss is examined for a configuration accounting for transverse particle diffusion in slender plasmas. The growth of the internal magnetic field (reduction of beta) leads to a significantly reduced confinement time compared to that predicted from the initial beta. Thus, while it may be possible to achieve a very high initial beta by staged laser solenoid heating, the average beta during confinement may be as low as 0.8 to 0.9. Particle end-loss may be greatly reduced by means of material end-plugs, in which case an extended energy confinement time may be achieved where the limiting loss mechanisms are bremsstrahlung and thermal conduction. Subject to certain minimum conditions on plasma length and radius, a steady-state burn can be achieved if cryogenic D-T end-plugs are used.

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FUSION REACTOR CONCEPTS BASED ON THE LASER SOLENOID*

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The feasibility of the laser solenoid reactor concept has been studied. In this paper we assess its plasma physics. In the laser solenoid the plasma is heated in stages by an axially directed laser beam, and magnetic field compression. Efficient, long-pulse, CO_2 lasers can be used which have a highly developed technology and demonstrated scalability. The plasma is confined radially by a linear solenoid magnet system composed of small bore magnets inside a large bore steady superconducting solenoid: this scheme offers a stable plasma configuration and a relatively straightforward engineering design.

Many pertinent reactor aspects were treated including: physics of the laser-plasma interaction; thermonuclear behavior of a slender plasma column; neutronics; radiation damage; and the design of the several key system elements. Self-consistent reactor configurations were developed for both pure fusion and fusion-fission hybrid designs. The fusion-fission designs are relatively small (100-300 m long with pulsed laser energies under 10 MJ), have low circulating power fraction ($f \lesssim 1/3$), enjoy relatively relaxed engineering constraints, and have attractive potential for near term application for fissile fuel breeding and/or electric power generation. The pure fusion designs are larger (1000 m long with laser energies of about 50 MJ), have higher circulating power fraction ($f \simeq 0.4$), and have tighter engineering constraints, particularly in the area of pulsed heating and stress in the first wall and pulsed magnet. Although the engineering development of such fusion reactors present major challenges, there is no technical reason why the laser solenoid could not be built and operated efficiently, and hence the concept is an attractive candidate for longer term commercial application.

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Double-Star Stellarator. T. S. Wang and T. H. Jensen, General Atomic Company--Stellarators for which the poloidal magnetic field has axisymmetric and nonaxisymmetric components (due to helical-like coils) have been investigated. For the cases examined, the nonaxisymmetric field is of high multipole order. Therefore, the rotational transform for the interior of the configuration is mainly caused by the axisymmetric part of the field. With properly chosen fields, it is possible to obtain configurations with large shear. Furthermore, by choosing an elongated cross section, the requirements on the toroidal magnetic field strength can be diminished. Finally, it is possible to find configurations with large magnetic wells.

The specific configuration considered is Double Star* with three magnetic axes. The magnetic field consists of a) a toroidal field, b) an axisymmetric quadrupole-like magnetic field due to toroidal currents in two coils, c) a vertical field, and d) a helical-like field due to currents in helical-like coils. The helical-like coils are all located on an axisymmetric surface with a rectangular cross section (a feature that facilitates fabrication of the coils).

Examples of computed "flux surfaces" for Double-Star configurations will be shown. These examples include configurations with a height-to-width ratio of 2.5 and aspect ratio of 13 and $q = 2.5$. Our examples also show how the vertical field (or a radial displacement of the quadrupole field coils) can produce a deep magnetic well. The shear is large in the interior of these configurations because $q \to \infty$ at the hyperbolic axis and in the exterior because of the high multipole order field. Therefore, the configurations are insensitive to field errors and may be able to confine plasmas with high "beta poloidal," Due to the elongated cross section, the ratio between the toroidal field strength and the poloidal field strength is considerably smaller (by about a factor of 3) than for a similar conventional stellarator with the same major radius, aspect ratio, and rotational transform.

*T. Ohkawa, Kakuyugo-Kenkyu 22, No. 6, 395 (1969).

MODELLING OF FUSION PLASMA IN MIRROR SYSTEMS

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For steady state operation of a mirror confined plasma, there is the need to determine: (1) the condition for buildup on a target plasma (2) the vacuum and wall requirements for sustenance, (3) the evolution of the density and temperature profiles (4) corresponding streaming plasma requirements for stabilization against DCLC modes and (5) effect of the streaming plasma on energy and particle confinement. A mirror injection test code is being developed to model such a system by including (1) multiple ion and neutral species and energy groups (2) finite gyroradius effects (3) spatial and temporal dependence (4) aimed neutral beams (5) atomic and molecular processes (6) self-consistent vacuum conditions due to surface effects (7) energy relaxation between charged particle species and (8) ad hoc scattering loss rates. Details of the model and organization of the code will be presented.

Poster Session 3A

Transport Processes

MULTISPECIES TRANSPORT IN COLLISIONAL TOKAMAKS

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We have developed expressions for the radial particle and energy fluxes appropriate to a multicomponent collisional tokamak plasma, in the large aspect ratio limit. This development constitutes a multicomponent generalization of the kinetic theory of Braginskii[1] and the hydrodynamic analysis of Rutherford[2], with the further generalization to include charge-exchange frictional effects on the radial fluxes. The calculation is subject to the conditions that the masses be disparate and that for each species the sum of the collision frequencies relative to the species in question and heavier species be large compared to the sum of collision frequencies with lighter species.

The Braginskii kinetic theory applies to systems in the collision-dominated regime, which in tokamak geometry corresponds to the Pfirsch-Schlüter regime. Extending it to several species, we show that its essential features continue to hold -- viz. each species can still be allowed to have a separate temperature, and the perturbation of each distribution function is coupled only to the hydrodynamical quantities of the heavier species. We find that the equation governing the perturbation of each species has the form of the equation considered by Braginskii, with an effective drift, collision frequency, and ionic charge. The solution yields expressions for the friction and heat conduction in terms of flow velocities and temperature gradients.

Specializing to a typical helical field, we solve for the leading terms of the radial particle and energy fluxes, both of which exhibit the Pfirsch-Schlüter enhancement. The particle flux is ambipolar. We show that the primary effect of neutrals entering the plasma from the walls is to produce an additional effective radial pressure gradient in these expressions. We discuss some typical situations to which our analysis applies.

(1) S. I. Braginskii, Soviet Phys. JETP 6, 358 (1958).
(2) P. H. Rutherford, Phys. Fluids 17, 1782 (1974).

The Fluid Picture of Neoclassical Diffusion*

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The two fluid equations for a plasma can be derived by taking moments of the Fokker-Planck equation. Therefore, they hold in all collisionality regimes. The reason that the diffusion and the plasma current behavior depend on the collisionality is that the parallel viscous force becomes nonnegligible in the plateau and neoclassical regimes. The parallel viscosity tries to maintain flow equilibrium along magnetic field lines - that is rigid body motion. In a symmetric torus, the parallel viscosity damps the poloidal rotation of the electrons, but not the toroidal. The radial diffusion produces a $v_r \hat{r} \times \vec{B}$ force, which in the Pfirsch-Schlüter regime is balanced by the force $en\eta\vec{j}$ associated with the resistivity. In the plateau and neoclassical regimes, $v_r \hat{r} \times \vec{B}$ is balanced not only by the resistivity, but also in the poloidal direction by the parallel viscosity. The toroidal current being more weakly damped than the poloidal is enhanced giving the bootstrap current. Using the two fluid equations with the parallel viscosity term, we have derived the well-known equations for plateau and neoclassical diffusion including the Ware drift term. These equations not only clarify the physical picutre of neoclassical diffusion, but also illustrate possible effects of anomalous collision frequencies.

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PFIRSCH-SCHLÜTER TRANSPORT IN MULTIPLE ION SPECIES PLASMAS*

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Impurities entering a tokamak plasma from the outside first encounter a plasma region that is collisional because of its low temperature. The impurities become ionized in this region but attain only low ionization states because of the low electron temperature. This edge region thus contains the source of impurity ions for the interior of the plasma. Consequently, the transport behavior of impurities there is very important because it determines the impurity feed rate to the interior of the plasma. Transport in the collisional or Pfirsch-Schlüter regime in the high impurity-ion-to-plasma-ion mass ratio limit has had both a fluid treatment[1] and a kinetic theory treatment.[2] However, this limit is inappropriate for low-mass impurities such as oxygen in a hydrogen plasma and such impurities appear to be common to most tokamak experiments. A kinetic theory treatment using a model operator has also been performed.[3] We have solved the particle and heat flux problems in the Pfirsch-Schlüter regime using a kinetic approach with exact Fokker-Planck collision operators. The previous treatments neglected the energy correlation of the cross-collision terms, which we find leads to a new temperature screening effect for low mass ratio, low-Z impurities. The coefficients of the pressure and temperature gradient terms which drive the fluxes are found to be functions of the impurity-ion-to-plasma-ion mass and charge ratios and the impurity strength parameter $\alpha \equiv_j \Sigma_{j,j\neq i} n_j Z_j^2 / n_i Z_i^2$. We present the results for a two-ion species plasma consisting of an oxygen impurity in a hydrogen plasma. In the appropriate limit we reproduce Rutherford's results for the driving term coefficients,[1] except for large α where temperature screening always occurs. Because of temperature screening the source of impurities for the interior of the plasma may be reduced, and this effect may be an important element in any impurity recycling model.

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Analysis of Neutrals in the TFTR Plasma Using Multigroup Transport Theory.†

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Multigroup transport theory is used to examine the interaction of the influx of cold neutral gas with a plasma characteristic of the TFTR[1] tokamak. It is found that the neutral particle densities in the body of the plasma are consistently lower than those predicted by the Duchs code[2] reaching a maximum discrepancy of over three orders of magnitude on axis. Of note here is that a reduced neutral population has been shown[3] to dramatically increase the figure of merit, Q, for a beam driven system.

Multigroup transport allows a detailed evaluation of the energy spectrum and angular distribution of outfluxed charge exchange neutrals. Half as many neutrals emerge, with a considerably softer spectrum than given by the Duchs code. Spectral information is of particular importance in calculating the influx of sputtered impurities.

The utilization of discrete ordinate transport modules in conjunction with cylindrical plasma transport codes is well within the state of the art and computationally efficient. Extensive cross section libraries have been created for use with the ANISN code[4]. The use of 20-30 logarithmically spaced energy groups is found to be sufficient to cover the range of neutral energies considered, consistent with the errors in the experimental data being fitted. Anisotropic scattering expansions have been shown to have little effect on the calculated neutral distributions, as have angular discretizations above S-8.

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Classical Diffusion: Equations and Numerical Algorithms

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The equations governing coupled "classical" (Pfirsch-Schluter, Neoclassical) diffusion and skin effect were formulated in 1969 by Grad and Hogan. In large aspect, low β limit, the solutions evolve on two time scales. This decoupling is not present at low β and finite aspect (e.g. in actual Tokamaks). The skin effect, even at low β and large aspect, is completely different in a deformable plasma from that in a solid conductor. An efficient numerical scheme for this limiting case has been exploited.[1] The general problem (finite β and aspect) is to put the equations in a form to "march" efficiently. This means: a) 1D diffusion of profiles and velocity is separated from occasional 2D evaluation of geometrical (inductance) coefficients; b) the equations explicitly exhibit second derivatives (diffusion) which are hidden in terms like $u\cdot\nabla\psi$; c) no variables that do not specifically diffuse are time-differenced; d) the fact that the determinant of coefficients of second derivatives vanishes at low β is explicitly exhibited (at low β only one magnetic field component diffuses).[1] The structure of a well-posed problem is subtle in a complex geometry (Doublet, tearing, Divertor) with a separatrix. The diffusive singularity at a separatrix is entirely different from that for the adiabatic problem and requires a new analysis of connection formulas. We choose the mass, M, within a flux contour as (1D) independent variable, Eulerian 2D variables, and take two fluxes and density as dependent variables. We describe why the innumerable other possible choices are less efficient.

[1] D.C. Stevens and E. Turkel, Diffusive Transition from Belt Pinch to Doublet, at this conference.

Investigations of Thermal Conductions Across a Magnetic Field

T. Kamimura, T. Wagner, and J. Dawson

We have investigated the transport of heat perpendicular to a magnetic field using a two-dimensional electrostatic particle model. At the left hand wall the temperature is maintained at a high value while at the right hand wall a low temperature is sustained. Initially the plasma is at the temperature of the cold wall. A temperature gradient is established with time and the heat fluxes carried by the electrons and ions are measured. A parameter study of the heat transport as functions of ω_{pe}/ω_{ce} (from 1/3 to 2) and ion to electron mass ratios (m_i/m_e from 1.25 to 9) has been carried out. In general the ion energy transport exceeds that for the electrons and is much greater than can be accounted for by ginary collisions. We believe the heat transport is caused by waves. As a consequence, we have made a study of the emission and absorption of Bernstein waves on a two-dimensional model. Initial damping followed by theoretically predicted recurrence is observed. The initial damping for long wavelength waves exceeds Landau damping for a field free plasma while the damping of the recurrence agrees with that calculated from convective modes. Observations of the recurrency and its damping could give a direct measure of the level of plasma turbulence.

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A TWO-DIMENSIONAL DIFFUSION TIME SCALE TOKAMAK CODE WITH NEUTRALS*

by

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G2M is a two-dimensional flux coordinate transport code designed to study the diffusion stage of the Tokamak discharge. Included in the code is the effect of neutrals on the evolution of the discharge. It is assumed that transport along the flux surfaces is sufficiently fast that transport across the flux surfaces only need be calculated. The transport problem is thus reduced to the solution of a set of coupled one-dimensional transport equations for the density n, the electron pressure P_e, ion pressure P_i, and safety factor q. The flux lines are determined by the simultaneous solution of the pressure balance equation $c\nabla P = J \times B$ with the set of transport equations.

To maintain the charged particle content of the plasma diminished by outflux through the boundary a two-group neutral model is included. An influx of cold neutrals is specified at the plasma boundary. Charge exchange in the plasma yields a hot neutral component and ionization of both species maintains the charged particle number. The neutral density profiles are calculated using two-dimensional asymptotic diffusion theory. Results of the effect of neutrals on PLT performance will be presented.

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TURBULENT TRANSPORT IN TOKAMAKS, ETC.

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A theory of turbulent transport based on taking velocity moments of the quasilinear Vlasov equation is used to calculate transport due to microinstabilities in a number of magnetic confinement schemes. Particular applications to be presented are

- Heating in tokamak systems due to trapped particle instabilities, compared with the usual estimates of heat conductivity due to these modes
- Enhanced resistivity in Tokamak due to trapped particle modes
- Bootstrap current in tokamak due to trapped particle modes
- Transport due to lower hybrid instabilities in sheared and favorably curved magnetic confinement systems, e.g., reverse field pinch and Tormac-like systems
- Late-time transport in theta pinch and Scyllac-type systems due to lower frequency drift waves.

A limited amount of material may be presented describing the formal basis of the theory, and some of the experimental verification of this approach.

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NUMERICAL STUDIES OF THETA-PINCH IMPLOSION DYNAMICS AND LONG-TIME ANOMALOUS TRANSPORT

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Results of three recent numerical studies of the implosion dynamics and post-implosion anomalous transport in theta-pinch plasmas are summarized in this paper. These investigations include: (a) Fluid-numerical studies of post-implosion anomalous transport. The collisionless fluid model of Liewer and Krall [1] has been extended to include the long-time transport effects associated with lower-hybrid-drift instability for low drift velocities ($V_E \lesssim v_{Thi}$). The system is initialized close to a two-fluid equilibrium with $r_{Li}/\Delta L \lesssim 1$, where r_{Li} is the ion Larmor radius and ΔL is the density gradient length scale. Resistivity due to the instability causes a decrease in cross-field current, and a corresponding broadening of the sheath and interpenetration of plasma and magnetic field. For a wide range of parameters, the measured increase in sheath thickness agrees well with the theoretical estimate $(\Delta L/\Delta L_0)^4 = 1 + 4V_D(t=0)t/\Delta L_0$, where $V_D(t=0) \simeq (\eta_{AN}c^2\beta/8\pi\Delta L_0)$ is the characteristic diffusion velocity. (b) Hybrid (Vlasov ions and fluid electrons) numerical studies of collisionless theta-pinch implosions. Recent numerical results show, for parameters appropriate for the Maryland High Voltage Theta Pinch, that reflected ions are produced in high-density ($2\times10^{13}\mathrm{cm}^{-3}$) and low-density ($4\times10^{12}\mathrm{cm}^{-3}$) antiparallel-bias runs, but not in low-density parallel-bias runs. The numerical results are in good agreement with recent experimental observations of ion dynamics. (c) Fluid-numerical studies of the influence of neutral-particle transport and ionization mechanisms on implosion behavior. In recent numerical studies, it is found that for parameters appropriate to Scylla 1-B (∿15% initial ionization) two-stage ionization [2] processes enhance the effective ionization rate by at least a factor of two (over impact ionization from the ground state). As reported previously [3], the relative importance of anomalous and classical transport processes depends on radial location, density, bias field, etc.

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Computer Simulations of Current Penetration and Anomalous Resistivity

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A two and one half dimensional magnetostatic particle model with vacuum boundary conditions is used to investigate current penetration and anomalous resistivity in a magnetized plasma. A current parallel to the external magnetic field is turned on gradually at the boundaries and the induced plasma current across the system is followed in time. Simulation results show that there are three stages in the time evolution of the current distribution. At early stages the current profile closely follows the classical skin structure. This is followed by a stage in which the imposed current density at the outer region is sufficiently large to excite microinstabilities; during this stage there is substantial anomalous resistance and the current penetration into the discharge is enhanced. The microinstability spectrum concentrates about the lower hybrid frequency. In the final stage, pinching occurs when the magnetic pressure generated by the current exceeds the pressure of the plasma plus that of the initial B field. The compression of the plasma causes further flattening of the current profile.

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MONTE CARLO CALCULATION OF COLLISIONAL EFFECTS IN A MIRROR-CONFINED PLASMA*

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A Monte Carlo method has been developed for computing the effects of coulomb collisions in a steady-state plasma. The main purpose of developing the technique is to study loss rates at the boundaries of mirror machines. The method complements the more customary finite-difference solution of the Fokker-Planck equation.

The computation proceeds by performing a random walk in velocity space. Steps are taken in speed v and in μ, the cosine of the polar angle θ. The quantities which determine the mean and variance of the step probability distribution are the drag and dispersion coefficients $\langle \Delta v_{\parallel} \rangle$, $\langle \Delta v_{\parallel}^2 \rangle$, and $\langle \Delta v_{\perp}^2 \rangle$, which are the same as those appearing in the Fokker-Planck equation. At present, the coefficients are obtained by assuming that the scattering distributions are Maxwellian; however, this can be generalized.

In numerical tests without losses, a Maxwellian particle distribution at the appropriate temperature is obtained, as required. In a practical application, the energy distribution of electrons escaping the mirror and fan regions into a direct converter has been computed.

Advantages of the Monte Carlo method include: Development of intuitive understanding of the physical processes, ease of treating complex loss boundaries in velocity space, and flexibility regarding the relative importance assigned to calculation in different regions in velocity space. On the other hand, the method will probably never compete with finite-difference methods in solving time-dependent problems. However, it is anticipated that Monte Carlo techniques may be practical in problems of high dimensionality, as has been the case in photon and neutron transport.

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Poloidal Profiles During Turbulent Heating*

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We wish to model the current penetration stage of a turbulently heated tokamak. The basic formulas can be written in the slab geometry since the dominant anomalous transport has a characteristic frequency much larger than the bounce frequency. Thus the basic framework is provided by the Maxwell and multi-fluid equations, with classical and anomalous transport. Displacement current in the y and z directions (poloidal and toroidal directions) are duly neglected. Quasineutrality is used. We show that the anomalous collision frequency dominates over the anomalous viscosity and thermal conductivity, and that the convective wave transport can be neglected. For the numerical estimates above we use the leading term in the quasilinear series. During the current penetration stage the distribution function for the particles will depart from a single Maxwellian type. Thus our first objective has been to compare numerically calculated poloidal magnetic field profiles with measured, published poloidal profiles. (The poloidal magnetic field has been calculated by using a code which handles the anomalous collision frequency self-consistently.) The agreement is good and we conclude that the current penetration stage can be satisfactorily described by this model. Theoretical temperature profiles in which the distribution function is fitted to a Maxwellian are considered, as well as effective temperatures in which the distribution function is fitted to an addition of Maxwellians. Preliminary comparisons with the experimental data are discussed.

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Neutral Transport from Classical Processes*

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One of the necessary constituents of any steady state Tokamak discharge is the background gas (atomic hydrogen), having substantial effects on both the particle and energy balance of the plasma. The classical process of inward neutral transport consists of a charge exchange random walk subject to losses by ionization. A kinetic theory for this process is given, including the effects of density profiles, geometry, and the distribution in ion velocities. Plasmas are characterized by the optical depth parameter

$$\mathcal{J} = \int_0^a dr \left(\frac{\nu_I + \nu_{cx}}{V_i}\right)$$

(where ν_I and ν_{cx} are ionization and charge exchange rates and V_i is the ion thermal velocity). Separate expansions are given for both the low density ($\mathcal{J} \ll 1$) and high density ($\mathcal{J} \gg 1$) regimes, culminating in analytic expressions for the neutral density. The results are compared to the observations of the Alcator experiment in the high density regime.

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Classical Diffusion in Toroidal Plasmas

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Time dependent diffusion in axisymmetric toroidal plasmas with finite aspect ratio is examined by using

$$\nabla P - J \times B = -\varepsilon^2 [\rho \frac{du}{dt} + V] \quad (1)$$

$$\frac{dP}{dt} = A - \rho a^2 \nabla \cdot u \equiv Y, \quad \frac{\partial B}{\partial t} = \nabla \times (u \times B - \eta J) \equiv Z \quad (2)$$

where A = entropy production due to resistivity and heat conduction and V = viscous force. The velocity u is determined from (1) by dropping the ε^2-term and using (2):

$$\frac{\partial}{\partial t}(\nabla P - J \times B) = \nabla (Y - u \cdot \nabla P) - J \times Z - (\nabla \times Z) \times B = 0 \quad (3)$$

which shows the velocity component u_ψ normal to the flux surfaces is governed by a non-homogeneous integro-differential equation

$$\nabla \cdot (R^{-2} \nabla \xi) + Q\xi + R^{-2} [T_1 \langle \xi / R^2 \rangle ']' + [T_2 \langle \xi \rangle ']' = I \quad (4)$$

where R = radius from major axis, primes denote ψ-derivatives, $\xi = RB_p u_\psi$, B_p = poloidal field and Q, T_i, I are given in terms of the instantaneous equilibrium state and $\langle \ \rangle$ denotes integration over the flux surface. The poloidal and toroidal velocity components are found from (3) to be

$$u_p = B_p F_2 + T_3, \quad u_t = B_t F_2 + RF_1 + T_4. \quad (5)$$

where T_3 and T_4 are given in terms of instantaneous equilibrium state while $F_1(\psi,t)$ and $F_2(\psi,t)$ must be considered as part of the initial data and their time evolution is determined by the ε^2-terms in (1). Evolution of magnetic field, plasma pressure and density are also described in detail, by using (2) and the continuity equation.

This theory handles non-isothermal finite-β plasmas. The details of these calculations will appear soon in the Physics of Fluids.

ORBIT-AVERAGING EFFECTS IN MIRROR CONFINEMENT SYSTEMS*

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ABSTRACT

The non-linear kinetic equation describing the evolution of the ion distribution for a mirror-confined plasma is averaged over the ion orbital bounce-motion along field lines. The result is a modified two-dimensional velocity-space Fokker-Planck equation. The variation of the magnetic field strength B(Z) with axial position Z is an essential new feature of this model as opposed to the magnetic square well model. A numerical bounce-averaging procedure has been incorporated with previous numerical work on the Fokker-Planck equation[1] to produce a new computer code for studying mirror confined plasmas. Our model includes all classical Coulomb processes as well as ionization sources and charge-exchange losses associated with neutral beam injection. Both transient and steady-state plasma behavior can be simulated.

For steady-state plasmas in reactor applications one of the most important figures of merit is the particle confinement parameter $\langle n\tau \rangle$. We find that $\langle n\tau \rangle$ values obtained from our new bounce-average model do not differ appreciably from those produced by the old magnetic-square-well model for cases studied thus far.

Transient plasma behavior in the 2XIIB experiment is also being studied with this new model. Axial density profiles and plasma decay characteristics are obtained for typical experimental operating conditions. High-beta modifications of the magnetic well can significantly affect the results.

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RESONANCES IN BINARY CHARGED-PARTICLE COLLISIONS IN A UNIFORM MAGNETIC FIELD

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Previous treatments of the binary collision problem in a magnetic field have been approximate, to the extent that only the maximum impact parameter is altered from the Debye radius λ, to an appropriate average Larmor radius ρ, whenever $\rho < \lambda$, thus resulting only in small changes in the Coulomb logarithm.

In this work we have solved the two-body problem in a uniform magnetic field. For the case of like particle collisions we find that a resonance between the velocities transverse to the magnetic field of the test particle ($\vec{v}_{\perp t}$) and field particle ($\bar{v}_{\perp f}$), $\bar{v}_{\perp r} = \bar{v}_{\perp t} - \bar{v}_{\perp f} = 0$, reduces the problem to that without a magnetic field as far as the impact parameter, scattering cross-section and relative velocity scattering angle are concerned, for small as well as large angle scattering. This resonance condition, however, results in large changes in the force of friction and diffusion coefficients as compared to the Spitzer values. For example the like-particle collision frequency for small test particle velocities turns out to be four times the Spitzer value. For large test particle velocities all collisional parameters are reduced by the factor, $\exp-(v_{\perp t}/v_{\perp o})^2$, compared to the Spitzer values, where $v_{\perp o}$ is the thermal velocity transverse to the magnetic field.

For the general case of $q_t = \mp q_f$, $m_t \neq m_f$, resonances have been obtained which decouple the relative velocity (reduced mass) motion from the center of mass motion. These resonances also lead to limiting cases where the problem reduces to the case without magnetic field as well as other cases where the impact parameter is naturally found to be equal to the Larmor radius of the field particle, or the test particle.

The author is indebted to Ira Bernstein for instruction on Lagrangian mechanics and to Niels Winsor for useful discussion.

NEOCLASSICAL TRANSPORT IN THE EBT DEVICE*

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Single particle confinement in the Elmo Bumpy Torus depends on the compensation of toroidal drifts by grad B x B and E x B drifts (produced by gradients in the mirror fields and by the radial ambipolar electric field). Transport of particles and energy across field lines occurs when scattering randomly moves particles onto orbits which are slightly less well-centered than those previous to the scattering event. Numerical calculations have been made of neoclassical particle transport employing two different techniques:

1) Transport coefficients are obtained from the drift kinetic equation (with Landau collision operator) using a variational method. These coefficients show a strong dependence on the parameters $e\phi/kT$ (ϕ = radial potential) and ν/Ω (ν = collision frequency, Ω = poloidal drift frequency). Examination of the first order perturbed distribution function indicates that the predominant diffusive losses occur at values of $v_{\parallel}/v$ near unity where the poloidal drift frequency Ω is small. Using transport coefficients similar to these and an ad hoc ambipolar electric field, radial density and temperature profiles are computed from one-dimensional, fluid equations for EBT.

2) The distribution function is computed directly using a finite difference technique to solve a pitch angle averaged, drift kinetic equation (with Fokker-Planck collision operator) in one radial and one velocity coordinate. Results show that the component of the perturbed distribution function contributing to the neoclassical diffusion rate is decreased by the ambipolar potential.

The understanding of particle and energy transport is an important step in modeling spatially resolved results which should become available from the EBT experiment in the near future as well as in extrapolations ahead to future devices.

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+Consultant from the University of Tennessee.

MAGNETIC FIELD RIPPLE EFFECTS IN CIRCULAR AND NONCIRCULAR TOKAMAKS*

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The nonaxisymmetric toroidal magnetic field ripple due to the discrete coils can be an additional trap for particles in a tokamak, causing them not to experience the rotational transform and hence to drift vertically. For the background plasma, collisions are usually strong enough to scatter the particles out of the ripple trapping region in velocity space before they drift out of the plasma. Thus, this effect leads only to an enhancement of the transport coefficients.[1] For particles trapped in the toroidal field, the average location of their banana orbits can experience a new radial drift due to the presence of ripples, which results in an additional diffusion.[2] The ripple induced transport coefficients have been calculated analytically for tokamaks with arbitrary cross-section and evaluated numerically in particular cases. It is found that for a given coil design the noncircular plasma might be subject to more severe ripple requirements than the circular plasma.

At the higher ion energies typical of neutral beam injection into large devices ($\gtrsim$ 100 keV), or alpha particles produced from D-T fusion (3.5 MeV), collisional effects are much weaker and the fast ions trapped in the ripples can drift out of the plasma before collisional detrapping occurs. We estimate analytically the direct loss for particles born within the ripple loss region and the subsequent loss due to particles diffusing into the ripple loss region. Since the estimated loss can be quite significant, we will discuss means to minimize it. The relaxation of toroidal flow due to field ripple will also be discussed.

*Research sponsored by the U. S. Energy Research and Development Administration under contract with Union Carbide Corporation.

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A Spline-Galerkin Approach to The Numerical Solution of Radial Transport Equations

J. C. Wiley

A computer code has been developed to solve systems of coupled parabolic partial differential equations using a Spline-Galerkin extrapolated Crank-Nicolson scheme. The transport equations are treated in weak or Galerkin form with the approximating solution space chosen to be a subspace of the piecewise polynomial space. Specifically the trial functions are chosen to be cubic splines, which requires the solutions to have continuity up to the second derivative and provides formal fourth order accuracy in the knot spacing. The knots are chosen dynamically to minimize the estimated interpolation error. The time integration uses both a predictor-corrector and an extrapolated Crank-Nicolson scheme with a dynamic time step control.

This code has been used to solve the Ware-Hazeltine neoclassical transport equations. The effects of the electrostatic trapping terms were investigated for a TFR type discharge by computing solutions with and without the electrostatic terms. Without the electrostatic terms, the equations reduce to the usual neoclassical equations in the plateau regime.

*Poster preferred.

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Poster Session 3B

High-Beta, Pinches, Plasma Simulation

and Boundary Layer Theory

High Beta Mirror Equilibrium Calculations for Baseball and Ying-Yang Configurations*

by

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ABSTRACT

A vector potential equilibrium code, VEPEC, has been built to obtain realistic $\underline{P} = \underline{P}(B,\psi)$ equilibria for 3D confinement devices with quadrapole symmetry. A major application has been to produce high beta equilibrium fields for use by orbit codes which follow the trajectories of representative particles. Some earlier attempts to follow orbits in equilibrium fields failed because the interpolation formulae for evaluating the magnetic fields did not preserve $\nabla\cdot\underline{B} = 0$. The near method which we use is to interpolate vector potentials and obtain $\underline{B} = \nabla \times \underline{A}$ analytically from the interpolation formulae; this guarantees $\nabla\cdot\underline{B} = 0$ to computer roundoff. For this reason the code solves the equilibrium in terms of the vector potentials. The pressure model used is the same as discussed by Hall and McNamara[1]. No assumption is made regarding open field lines so reversed field mirror equilibria can be sought with the code. The requirement of quadrapole symmetry could be removed in a future version of the code which would allow representation of general non-symmetric 3D confinement schemes including stellarator, PDX, and other realistic devices. Results from the orbit codes, showing adiabatic and super adiabatic confinement in high beta fields, will be presented.

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NONLOCAL PROPERTIES OF CURRENT-DRIVEN CROSS-FIELD INSTABILITIES IN HIGH-DENSITY PINCHES*

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Analytic calculations, computer simulation experiments, and laboratory experiments all indicate that cross-field instabilities (lower-hybrid-drift, Buneman, etc.) play an important role in the implosion heating and anomalous transport in high-density pinches. Previous theoretical studies of these instabilities have been based on the local approximation, without systematic justification a posteriori. This paper summarizes several recent studies of the nonlocal properties of cross-field instabilities. These investigations include: (a) Analytic and numerical studies of the radial eigenvalue equation obtained within the framework of a macroscopic two-fluid description of theta-pinch plasmas with $T_i >> T_e$ and $V_E >> v_{Thi}$. The limiting form of the lower-hybrid-drift eigenfunctions and growth rate are examined for the (nonlocal) case of sharp-boundary equilibria. Moreover, for sufficiently diffuse equilibrium profiles, nonlocal and local theories are shown to be in excellent agreement for the fastest growing mode in the system. Using experimental profiles, the lower-hybrid-drift instability is also shown to be a viable mechanism for predicting the wavelength and radius of localization of the flute perturbations observed in Scylla 1-B. Finally, the macroscopic two-fluid model is generalized to linear screw pinch geometry, and the influence of magnetic shear on nonlocal stability properties is discussed. (b) Studies of kinetic stability behavior. A fully kinetic, nonlocal, matrix dispersion equation is derived for electrostatic perturbations about a cylindrical rigid-rotor Vlasov equilibrium. Assuming equilibrium charge neutrality and uniform axial magnetic field, it is shown that the particle trajectories and the required orbit integrals can be evaluated in closed form. The analysis leads to a matrix dispersion equation of the form $\det[\delta_{n',n}+\sum_j \chi^j_{n',n}(\omega+i\gamma)] = 0$, where the susceptibility $\chi^j_{n',n}(\omega+i\gamma)$ is expressed as a phase-space integral over $f^0_j(\underset{\sim}{x},\underset{\sim}{v})$ and known functions of ω, r, etc. This matrix dispersion equation is simplified analytically and solved numerically in a parameter regime appropriate to the lower-hybrid-drift instability ($\omega_{ci} << |\omega+i\gamma| << \omega_{ce}$), and detailed comparisons between local and nonlocal theory are presented.

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THREE DIMENSIONAL ANISOTROPIC PRESSURE EQUILIBRIA IN MIRROR MACHINES*

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ABSTRACT

Anisotropic equilibrium and stability is investigated in the guiding center fluid model. For the omnigenous 2XII and Baseball fields a mapping exists between the drift potential and pressure weighing function along a radial ray. Thus, the magnetostatic formulation of Hall and McNamara[1] determines an equilibrium from the solution of Poisson's equation and several magnetic differential equations.

The McGUS code solves the equations for finite β fields used to investigate stability, particle orbits and spatial diffusion. Fig. E1 shows plasma field lines spiralling round the plasma current. Fig. E2 shows the total field structure at β = 25% in 2XII. Fig. E3 shows the deepening of the magnetic well in 2XII by the plasma. Plasma stability is monitored by standard criteria and by observation of

$$\overline{\Gamma} = \int \frac{\hat{P}_\parallel d\ell}{|B|}$$

which must be radially monotonic for stability and also $\hat{P}_\parallel(\vec{k}\cdot\nabla\omega)$ which is related to a destabilizing term in the energy principle.

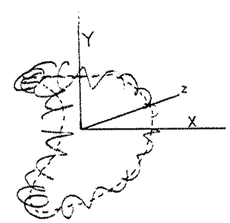

Fig. E1

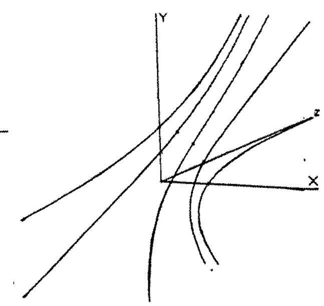

Fig. E2

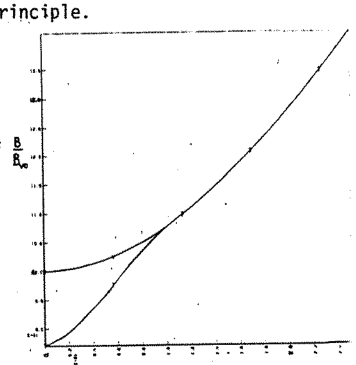

Fig. E3

*Work performed under the auspices of the U.S. Energy Research and Development Administration under contract No. W-7405-Eng-48.

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Conversion of the Three-Dimensional Sheet- or Screw-Pinch Stability Problem to a One-Dimensional Simulation

by Oscar Buneman, Stanford University*

In principle, screw and sheet ("belt", "doublet") pinch Vlasov-stability tests can be reduced to an eigenvalue search.[1,2,3] In practice such a search is very computer-expensive: no verdict on the existence or non-existence of Vlasov-stable screw or sheet pinches has yet been achieved. The equilibria have three ignorable co-ordinates (Θ,z,t or y,z,t), but rather than transforming all three in the perturbation analysis (to m,k,ω or k_y,k_z,ω), we now only transform the two ignorable space co-ordinates and obtain the time-evolution by a one-dimensional computer simulation. This necessitates an Eulerian treatment of the particles: we use stream superposition, classifying streams by their unperturbed, canonical momenta[3] L,P,E, and time-stepping stream potentials $\psi(r,L,P,E)$ and stream densities $f_1(r,L,P,E)$. A second-order space-time staggered-grid leap-frog scheme results from the Hamiltonian[2] nature of the problem. Housekeeping in the partially filled (x,L,P,E)-space is difficult but has been resolved. First runs pertaining to the ply-wood sheet pinch[4] (current filaments sheared) are yielding results.

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NONADIABATIC EFFECTS IN FINITE BETA MIRROR MACHINES*

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ABSTRACT

Finite β has a deleterious effect on adiabatic confinement of particles in mirror machines since magnetic field strengths and scale lengths are reduced. Analytic techniques described previously for calculating magnetic moment jumps in mirror machines[1] and simpler magnetic configurations[2,3] have been applied to axisymmetric model B fields representing finite-β mirror equilibria. The results have been compared to those obtained from particle orbit codes and are found to be in good agreement.

Analytic calculations have been done for equilibria determined analytically from the long, thin plasma approximation and for model fields approximating results of a 3-D equilibrium code. Particle orbits have been run for the analytic models and the computed equilibria. Axial pressure distributions which extend most of the way to the mirrors (as in a collisionally dominated plasma) as well as narrow profiles (such as in 2XIIB) have been considered.

The energy at which the nonadiabatic loss rate neglecting gyrophase correlations (superadiabaticity) becomes comparable with the loss rate for all other processes has been estimated. For the most axially spread pressure profiles chosen, the limiting energy is reduced by only about a factor of 2 as β changes from 0 to .5; for narrow profiles, it is reduced by more than an order of magnitude. Numerical and analytical results both indicate the presence of superadiabatic effects at energies exceeding this limit. If gyrophase correlations are not destroyed by other processes, a higher energy limit results.

Results for parameters appropriate for 2XIIB and future mirror experiments are discussed.

*Work performed under the auspices of the U.S. Energy Research and Development Administration under contract No. W-7405-Eng-48.

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DIFFUSE SCYLLAC-LIKE EQUILIBRIA*

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Diffuse, Scyllac-like equilibria are sought assuming an azimuthally varying lowest order magnetic field $B(r,\theta)$. The original set of MHD equilibrium equations is reduced to two equations involving a stream function ϕ, $(B \equiv \nabla\phi/B)$, the kinetic to magnetic pressure ratio $\beta(r,\theta)$ and an arbitrary function $g(\beta)$. The low β, stellarator like, solutions can be obtained by assuming $\beta << 1$ and by suitably choosing $g(\beta)$. A high β limit expansion is also performed and various analytic forms for $g(\beta)$ are studied in this limit. A toroidal force balance is indeed possible in the same limit.

*Work performed under the auspices of the U.S. ERDA.

COMPUTER STUDIES OF THE INJECTION AND COMPRESSION OF STRONG ION RINGS*

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As part of the continuing studies at Cornell of the properties of strong ion rings, we are studying the formation and trapping of ion rings by the injection of intense ion beams into various magnetic field configurations. The code utilized treats the beam or ring ions by a volume-weighted finite-size particle-in-cell method in two dimensions (r and z). The displacement current is neglected and only the azimuthal component of the vector potential is retained, and the background plasma is assumed to be present in order to provide quasi-neutralization of charge and of currents in the r- and z-directions. In order to be able to include effects due to the plasma electrons without the restrictions imposed by the very rapid electron time scale, a simple fluid model based on Ohm's Law is used to calculate the azimuthal electron current and first-order electric field.

Aspects of the injection and compression processes which are studied include the following: 1. Compression of rings by an adiabatic temporal increase in the magnetic field. Self-similar compression beyond field reversal on axis has been studied, and we find BR^2 ~ constant and field reversal factor $\zeta \sim B^{1/2}$, as expected. 2. Injection of an annular beam through a non-adiabatic magnetic cusp. Good transmission is achieved without loss of particles. 3. Injection of such rings into a spatially increasing magnetic field, and reflection by a magnetic mirror field. 4. Slowing down of the axial motion of rings using resistor wires for trapping in mirror fields.

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THE LOWER HYBRID DRIFT INSTABILITY AT LOW DRIFT VELOCITIES*

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In recent years the lower hybrid drift instability has received considerable attention in connection with high β pinches where $T_i \gtrsim T_e$. There is a general consensus that this instability plays an important role in broadening the plasma sheath during the implosion phase, $t \lesssim 1$ μsec. Perhaps more importantly, there is considerable concern that this instability could lead to anomalous transport on the long, post-implosion, time scale $t \gtrsim 100$ μsec, since early calculations indicated that there was no clearly defined lower threshold in drift velocity for the instability.

We report here on two calculations which address the lower hybrid drift instability in the slow time scale, low drift velocity regime. The first is a recomputation of the linear theory in which we treat the ions with circular magnetized orbits whereas the original calculations considered only straight line ion orbits. The magnetic field becomes important for the ions when ω becomes on the order of ω_{ci} as it does for sufficiently low drift velocities. We find that at low drift velocities the mode becomes resonant in nature and is stabilized when

$$\frac{V_D}{V_{Ti}} \le 4.18 \left(\frac{m_e}{m_i}\right)^{1/2} \left[1 - .651 \left(\frac{m_e}{m_i}\right)^{1/4}\right]$$

The second calculation is a new estimate of the nonlinear resistivity when the instability persists. The new feature here is the use of ion trapping, $\omega_{\text{bounce}}^{\text{ion}} \approx$ linear growth rate, as the saturation mechanism (as indicated by some of our plasma simulations) for determining the amplitude of the field fluctuation energy. In the bulk of the pinch, this gives a much reduced estimate of anomalous resistivity

$$\eta = \left[\frac{\pi^3}{2^9}\sqrt{\frac{\pi}{2}}\right]\left[\frac{v_{Di}}{v_{thi}}\right]^8 \frac{\omega_{ce}^2}{\omega_{pe}^2}\,\frac{1}{\omega_{LH}}$$

versus the original estimate in which Fowler's upper bound was used to calculate the amplitude of the fluctuation field energy.

*Work performed under the auspices of the U.S. ERDA

STABILIZATION OF m ≥ 2 MODES IN DIFFUSE HIGH BETA PINCHES*

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The stabilization of $m \geq 2$ modes in high β pinches, in particular Scyllac, has been investigated. By using a combination of theoretical analysis and numerical simulation techniques it has been possible to obtain fairly good estimates of the finite gyroradius stabilization criterion in diffuse high β pinch configurations. All previous calculations were concerned with sharp boundary, surface current models.

The calculation is carried out in the following way. The model used is the Vlasov-fluid model (i.e., Vlasov ions, fluid electrons described by $\underline{E} + \underline{U}_e \times \underline{B} = 0$). The equations are linearized and then cast in the form of an energy principle. Because of the close connection between the Vlasov-fluid model and ideal MHD, the eigenfunction of the ideal MHD system is an excellent trial function to use in the energy principle. The first step in the calculation is the evaluation of the ideal MHD eigenvalue and eigenfunction for a high β diffuse $\ell = 1$ system for the $m \geq 2$ modes. This in itself is fairly complicated and is accomplished by a combination of analysis, utilization of the "old" Scyllac expansion and numerical computation.

This trial function is substituted in the Vlasov-fluid energy principle. The frequency ω, enters the energy relation in a highly transcendental way and must be determined iteratively, each iteration consisting of the evaluation of a multidimensional (i.e., in our case, four) integral along the complicated unperturbed orbits of the particles. The evaluation of these integrals is carried out using particle simulation techniques.

A discussion of the method and results will be presented. One important result is that the stabilization criterion is significantly more optimistic than the early sharp boundary predictions.

*Work performed under the auspices of the U.S. ERDA.

COMPUTATION OF PLASMA-WALL INTERACTION IN A TOROIDAL THETA PINCH

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ABSTRACT

Using the two-dimensional computer code ANIMAL -- A New Implicit Magnetohydrodynamic ALgorithm [1,2] — we have computed in both one and two dimensions the long-time behavior of an infinite "toroidal theta pinch". In the one-dimensional case, after the plasma experiences the toroidal drift, a cold dense layer adjacent to the wall is formed as the plasma, initially cooled by thermal conduction, undergoes a "radiation collapse" when bound-free and bound-bound radiation becomes dominant. In the two-dimensional case, the cold dense layer again forms, after which a thermal-conduction/radiation induced instability occurs, leading to violent swirling and enhanced cooling of the plasma. The existence of such instabilities due to contact with a wall would apppear to have important ramifications in fusion proposals where the hot plasma is confined by a wall or cold gas. The computations are displayed in a computer-generated movie.

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Submitted for presentation at Annual Meeting on Theoretical Aspects of Controlled Thermonuclear Research, University of Wisconsin, Madison, Wisconsin, April 7-9, 1976.

Because this paper requires movie projection, it is requested that it be selected for an oral session and not a poster session. Contact: I. R. Lindemuth, 415-447-1100, Ext. 7311.

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COMBINED BETA AND ANISOTROPY LIMITS SET BY ALFVÈN-ION-CYCLOTRON MODE*

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ABSTRACT

For the Alfvèn-ion-cyclotron (AIC) mode, the purely temporal behavior (complex ω for real k) has been discussed many times.[1,2] We have looked at the combined spatio-temporal behavior[3] (complex ω, complex k) of AIC disturbances. In particular, we have located the transition from convective to absolute growth[3] which occurs as one increases β or increases anisotropy ratio. The resulting convective-absolute boundary curves are displayed on a $\beta_\perp - \beta_\parallel$ diagram for bimaxwellian, loss-cone-maxwellian and Holdren[4] velocity distributions.

Spatial variation along the magnetic field, assumed quadratic, yields WKB corrections[5] to the complex frequency. Favorable shifts in the convective-absolute boundary result.

Spatial variation across the magnetic field provides an effective $k_\perp^2$ which we insert into the homogeneous dispersion tensor. Again growth rate corrections shift the absolute AIC instability to higher β-values.

Our ongoing work concerns realistic velocity distributions derived from Fokker-Planck codes.[6] These are approximated by a maxwellian times an analytic function of angle whose height and width are arbitrary functions of velocity. Dispersion function evaluation for this case is described briefly.

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*Work performed under the auspices of the U.S. Energy Research and Development Administration under contract No. W-7405-Eng-48.

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Results of Computer Experiments with a MHD Code*

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We are employing the Trinity MHD code of Boris and Roberts for studies of MHD stability, nonlinear behavior, and wave propagation and for comparison with the results of particle codes. This code is based on ideal MHD but allows for finite resistivity and viscosity. The MHD equations are integrated using an explicit difference scheme and an Eulerian mesh. At present we have implemented the scheme for Cartesian coordinates in two and one-half and three dimensions. The 2½-D code with periodic boundary conditions has been used to study propagation of Alfven waves in nonuniform plasmas. The predicted damping of these waves due to phase mixing is observed. The 3-D code employs rigid, conducting walls in the x and y directions and periodic boundary conditions in the z direction. Stability questions, including the possible generation of reverse magnetic fields, are being investigated.

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**NSF Postdoctoral Energy-Related Fellow

Numerical Solution of the Axisymmetric MHD Equilibrium Equation*

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The Grad-Shafranov equation for axisymmetric systems is solved by finite difference methods for a variety of field profiles. Irrespective of the choice of profiles similar behavior is found in parameter families of $q_{min} = 1$ equilibria with diamagnetic toroidal field, as the poloidal beta increases. The magnetic axis shifts outward, the toroidal current density becomes sharply peaked on the outer edge of the plasma and reverses on the inner edge of the plasma, and eventually a maximum in the flux appears on the inner edge of the device. Similar behavior is found for increasing total beta. The betas are limited by stability considerations when the toroidal current becomes sufficiently large in the low density region at the outer edge of the plasma. Numerical and analytic estimates for the occurence of bifurcated solutions are discussed.

A finite element code for the solution of the equilibrium problem has produced results for bifurcated problems in good agreement with those of the finite difference method. For curvilinear boundaries the finite element method is capable of representing both the solution and the boundary conditions more accurately.

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Radial Boundary Layers in Diffusing Toroidal Equilibria*

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Analytic results in straight cylindrical geometry imply sharp density gradients near the boundary of a plasma decaying by classical diffusion. Utilizing an isothermal one-fluid MHD model, we apply these results to toroidal configurations and obtain a set of nonlinear equations for a radial boundary layer. A dominant effect in this regime is convective plasma flow along magnetic lines of force, with velocity in the sonic range. This flow pattern matches onto the interior solution of Pfirsch-Schluter convective flow. Solutions separable in the radial and poloidal directions are found which fulfill both boundary and periodicity conditions, and that result in both smooth subsonic poloidal flows, and weakshock transonic flows. The shock structure is solved, with resistivity η, and viscosity μ, as dissipative mechanisms, and the shock width is found to scale as $\sqrt{\mu\eta}$. Effects of the flow patterns on diffusion are discussed.

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A Sharp Boundary Dynamical Model for Scyllac

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The ideal MHD equations are linearized about a toroidal helical theta pinch equilibrium and by means of the "new Scyllac expansion" developed by Weitzner, the coupled equations of motion for gross plasma displacement and the various helical displacements are derived. Included in the analysis are external time dependent perturbing L = 0, 1, 2 magnetic fields which may be used in the feedback stabilization of the m = 1 instability. When the determinant of the system is analysed one finds the usual m = 1 growth rates for the L = 0 and L = 2 toroidal equilibrium fields. When both L = 0 and L = 2 fields are present, there are two unstable eigenmodes corresponding to growth rates in the toroidal (horizontal) and vertical planes. The relative amounts of L = 0 and L = 2 fields may be optimized to give neutral stability for the vertical eigenmode and consequently the horizontal eigenmode is larger than the average growth rate. The average growth rate may also be optimized; this corresponds to near neutral stability for the vertical eigenmode. This anisotropic effect can have important consequences on the design of feedback systems for an L = 0, 2 Scyllac, since the growth rates due to these fields are of the same magnitude as the growth rate due to the L = 1 field found in the old Scyllac expansion.